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**A POLICY EVALUATION MODEL AND  
COMPUTER-ASSISTED POLICY EVALUATION  
FOR NAVAL PERSONNEL MANAGEMENT**

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A POLICY EVALUATION MODEL AND PROTOTYPE COMPUTER-ASSISTED POLICY  
EVALUATION SYSTEM FOR NAVAL PERSONNEL MANAGEMENT

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This study develops a prototype Computer-Assisted Policy Evaluation (CAPE) system for solving Naval personnel assignment problems. The CAPE system utilizes a new mathematical formulation for multiobjective function assignment problems which is capable of evaluating a number of important personnel management problems. The computer program documentation for the CAPE system is included.		

#### FOREWORD

This study was in support of personnel assignment policy decision-making in the Bureau of Naval Personnel. The computer program discussed in the model has been implemented at NAVPERSRANDCEN for further evaluation. Special acknowledgement is due to Mr. John Malone who provided technical background and consultation on the problem. The technical monitor was Dr. Stephen W. Sorensen.

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## SUMMARY

### Problem

Naval personnel assignment policies either directly or indirectly affect almost every facet of naval operations. Therefore, such policies should not be relegated to ad hoc and rule-of-thumb procedures but should be given a thorough quantitative and qualitative analysis to determine effective decision alternatives.

### Objective

The primary objective of this study is to develop an analytical tool to help policy makers accurately determine the effect of alternative policies.

### Approach

The objective was accomplished through the following four steps:

1. Formulating a mathematical programming model expressly designed to resolve the complex interactions of the policy area.
2. Developing specifications for the use of this model in carrying out policy analysis.
3. Designing and programming a prototype Computer-Assisted Policy Evaluation System (CAPE) for Naval Personnel Management. This system includes the development of computer programs capable of generating and solving the proposed mathematical programming model for a subset of the Navy rate and rating groups, utilizing existing personnel and requisition data bases.
4. Conducting a preliminary computer evaluation study of policies related to: (a) maximizing the number of available and eligible men that can be assigned to unfilled billets regardless of utility, cost, or desirability attributes, (b) developing criteria for weighting various components of manpower qualifications and job attributes, (c) determining optimal assignments of men to billets based on the weighting criteria, and (d) developing modelling capabilities for handling projected rotation dates.

### Results

A new type of mathematical programming mode, termed an extended goal programming (EGP) model, has been developed. This model provides insights into many policy questions and can be constructed using information currently available in the Enlisted Master file, requisition files, and student avail files. The EGP structure incorporates a methodology for studying questions in the policy area related to the fundamental cost, desirability, and utility functions. It allows the policymaker to specify what relationship is desired between the attribute functions and then to derive the appropriate weights in advance that will lead either to an equal representation of the components being weighted or a representation which is unequal by preselected ratios.



To apply the EGP model in the most effective manner with currently existing solution capabilities, a mathematical technique for obtaining good approximate solutions to the fundamental model structure has been developed. This technique is based on a form of dynamic sensitivity analysis utilizing a generalized Lagrangean relaxation. Basic to all stages of the model design and the solution methodology is a stepwise determination of appropriate algorithmic modifications to the computationally efficient network code.

A CAPE System was developed which provides the capability for enforcing desired proportionalities between conflicting objectives in manpower planning applications. This system adds a means of evaluating the effects of policies dealing with the number of personnel assigned to billets. For example, the influence of the preemptive policy of assigning the maximum number of personnel to billets can be isolated and the trade-offs between maximum personnel assignments and assignments that maximize other measures such as utility and desirability can be determined. Although the CAPE System presently encompasses only the Disbursing Clerk (DK), Aviation Maintenance Administrationman (AZ), and Hospital Corpsman (HM) ratings, it is generalizable to the other enlisted ratings.

The Computer-Assisted Policy Evaluation System has been evaluated by solving a number of problems and performing a variety of preliminary analyses. Three primary policy areas, each representing a distinct type of policy, were subjected to extensive testing. These areas involve (1) the multiattribute facet of the assignment process, (2) the preemptive fill policy, and (3) a major billet eligibility policy.

#### Recommendations

It is recommended that CAPE be used:

1. To test the effect of varying the formulas for calculating the parameters of the utility and desirability functions. In particular, the EGP model can be expanded to include each parameter as a separate function.
2. To test the effect of allowing the proportionality weights to be parameters in the model, so that policymakers would be provided with an idea of the influence of attaching different degrees of importance to each goal.
3. To test such critically important questions as whether the growth of NEC ratings should be stopped. This could be evaluated by grouping similar NEC ratings and using CAPE to solve the resulting problems.
4. To evaluate the billet fill priority system. This priority system includes preemptive priority classifications, namely a MUST FILL priority and priorities 1 and 2 assigned by the Chief of Naval Operations. The effect of these preemptive priority classifications should be determined since the NEOCS plan reported that these priorities substantially complicate efficient detailing of personnel.
5. To evaluate billet and assignment rotation eligibility policies.

## CONTENTS

	Page
INTRODUCTION . . . . .	1
Problem. . . . .	1
Purpose. . . . .	1
APPROACH . . . . .	3
GLOBAL POLICY EVALUATION MODEL . . . . .	5
Overview . . . . .	5
Model Description. . . . .	6
Decomposition of the Global Model. . . . .	8
Solution . . . . .	9
EXTENDED GOAL PROGRAMMING MODEL. . . . .	11
Approaches to Manpower Modelling . . . . .	11
Model Description. . . . .	13
SOLUTION APPROACH. . . . .	15
Constraint Accommodation . . . . .	15
Dynamic Sensitivity Analysis . . . . .	15
Preemptive Policy Evaluation . . . . .	18
PROTOTYPE COMPUTER-ASSISTED POLICY EVALUATION SYSTEM . . . . .	21
Computer Implementation. . . . .	21
Comparison of CAPE and CADA. . . . .	21
CAPE System Overview . . . . .	22
Input Interface Phase. . . . .	22
Model Construction Phase . . . . .	22
Optimization Phase . . . . .	22
Derivation of Lagrangean Step Size and Search Procedures . . . . .	25
COMPUTATIONAL TESTING AND EVALUATION . . . . .	27
General. . . . .	27
Multiattribute Assignment. . . . .	27
Preemptive Fill Policy . . . . .	34
Billet Eligibility Policy. . . . .	37
CONCLUSIONS AND RECOMMENDATIONS. . . . .	41
REFERENCES . . . . .	43
APPENDIX - CAPE 204 AND 205 PROGRAM DOCUMENTATION. . . . .	A-0
DISTRIBUTION LIST	

## INTRODUCTION

### Problem

Naval personnel assignment policies either directly or indirectly affect almost every facet of naval operations. For example, policies which determine candidate eligibility to fill billets and those which determine the actual procedural rules (priorities) by which assignments are made clearly have a major impact on the overall effectiveness of naval manpower in accomplishing essential task requirements. In addition, such policies have a vitally important influence on morale and job satisfaction. Thus, policies of such influence should not be made through ad hoc and rule-of-thumb procedures, but should be established only after performing a thorough quantitative and qualitative analysis to determine effective decision alternatives.

### Purpose

In view of the above, the primary purpose of this project is to develop an analytical tool to help policymakers accurately determine the effect of alternative policies. Such a tool can be used to derive policies that will attain the following objectives of naval personnel management:

1. Matching manpower requirements with available assets in a "best fit" (i.e., putting the right man in the right place at the right time and at minimum cost).
2. Making maximum use of skills and training that are possessed by naval personnel.

To accurately determine the effects of manpower management policies, it is necessary to consider the assignment of personnel to billets on a global basis. That is, all different policies must be considered simultaneously in order to derive their interactive effects. Therefore, the complexity of designing an effective evaluation tool is accentuated by the sheer numbers of personnel and billets that must be considered. This evaluation tool must further be able to handle attribute characteristics for both personnel (e.g., test scores, NEC codes, pay grade, number of dependents, rate, rating, preferred geographical location, current location, etc.) and billets (e.g., rate, rating, location take-up date, primary and secondary NEC requirements, etc.). To accommodate these extremely large numbers of interactive considerations--and the effect of these interactions on potential billet assignments--a sophisticated mathematical programming model is required. In addition, efficient computer routines are needed to generate and solve the mathematical programming model in order to be able to analyze alternative policies. Thus, the specific purposes of this project are:

1. To formulate a mathematical programming model expressly designed to resolve the complex interactions of the policy area.
2. To develop specifications for the use of this model in carrying out policy analysis.



3. To design and program a prototype Computer-Assisted Policy Evaluation System for naval personnel management. This includes the development of computer programs capable of generating and solving the proposed mathematical programming model for a subset of the Navy rate and rating groups, utilizing existing personnel and requisition data bases.

4. To conduct a preliminary computer evaluation study of policies related to: (a) maximizing the number of available and eligible men that can be assigned to unfilled billets regardless of utility, cost, or desirability attributes, (b) developing criteria for weighting various components of manpower qualifications and job attributes, (c) determining optimal assignments of men to billets based on the weighting criteria, and (d) developing modelling capabilities for handling projected rotation dates.

## APPROACH

To accomplish the foregoing objectives, we have developed a new type of mathematical programming model, termed an extended goal programming (EGP) model. This model provides insights into many policy questions and can be constructed using information currently available in the Enlisted Master file, requisition files, and student avail files. Extended goal programming is distinctly more sophisticated than the state-of-the-art assignment-transportation model structures (Charnes, Cooper, & Niehaus, 1973; Malone & Thorpe, 1973; Malone, Thorpe, Tate, & Pehl, 1974) currently being used. The EGP structure fully incorporates a methodology for studying questions in the policy area related to the fundamental cost, desirability, and utility functions. In contrast to the typical procedure of forming an a priori weighted combination of these attribute functions to obtain a single objective function, the more general model allows the policymaker to specify what relationship is desired between the attribute functions and then to derive the appropriate weight based on this relationship. Since there is no known way to determine weights in advance that will lead to an equal representation of the components being weighted or to obtain a representation which is unequal by preselected ratios, the advances afforded by the EGP model are extremely important.

To apply the EGP model in the most effective manner with currently existing solution capabilities, we have developed a mathematical technique for obtaining good approximate solutions to the fundamental model structure. This technique is based on a form of dynamic sensitivity analysis utilizing a generalized Lagrangean relaxation (Shapiro, 1971). Basic to all stages of the model design and the solution methodology is a stepwise determination of appropriate algorithmic modifications to the computationally efficient network code (described in Barr, Glover, & Klingman, 1974). Thus, the cornerstone of the solution effort is a specially tailored version of this network code.

The next section contains a brief description of a global policy evaluation model. This model sets the stage for the special model structures subsequently described by highlighting the important considerations that can be accommodated when data bases are expanded to include the necessary parameter information and when solution techniques are developed to solve problems of this magnitude. Following the outline of the global model, we present a detailed description of the special models and solution techniques developed and implemented under this contract for the current prototype computer-assisted policy evaluation system. As will be seen, this prototype computer system provides substantial advances to current policy evaluation procedures.



## GLOBAL POLICY EVALUATION MODEL

### Overview

Careful analysis of the mathematical models which have been previously developed for the study of problems related to balance, rotation, requisition, assignment, and weighted attributes (Borgen & Thorpe, 1967, 1970; Butterworth, 1973; Charnes et al., 1973; Malone et al., 1974) discloses major limitations to these models. In particular, none of the earlier models provides the capability to analyze the effects of policies on the global (i.e., by rate and rating) distribution and retention of naval manpower. The model used in CAPE has the same limitation, however it is important to characterize the essential components of a totally global, dynamic model which is capable of broad-level analytic evaluation of a policy and its effects. Although implementation of a global model is beyond the solution capabilities of present-day systems, the specification of its fundamental characteristics provides a useful frame of reference for identifying the direction in which modelling and solution techniques should be heading. This totally global model is an area-and-time phased transshipment network model whose advanced structure is capable of distinguishing categories of billets by major claimant, activity, and related class description. At the same time, the global model also accommodates various differentiated classes of personnel who may be candidates for available and forthcoming billets.

The global advanced planning and evaluation model is directly susceptible to the specialized dynamic sensitivity analysis and extended goal (multiobjective) programming techniques which constitute two of the principal development facets of our investigations. This fact makes the global model more than a theoretical guidepost for present evaluations by disclosing its attractiveness for real-world implementation once the appropriate data bases and extended solution methodology become available. However, especially relevant to our present concern is the fact that the global model provides the basic insights that lead to the EGP model and solution techniques that underlie the central portion of this study. The EGP model is more limited than the global model because it cannot fully accommodate priority assignment rules and balance rules. However, the EGP model goes beyond previous models in its ability to evaluate the imposition and relaxation of rate and rating policies related to such items as personnel eligibility for assignment and for a particular billet. This is important because our investigations indicate that rate and rating policies represent a major portion of personnel manpower policies. Additionally, the extended goal programming model accommodates the following critical policy questions:

1. How should the personnel/billet attributes be used in assigning personnel to billets on a rate/rating basis?
2. Should filling the maximum number of billets with available personnel preempt all other considerations?
3. What are the key trade-offs among attributes and targeted staffing levels that underlie optimum manpower assignments?

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4. How does the relaxation of current regulations affect these staffing levels and manpower assignments?

The Computational Testing and Evaluation section demonstrates how the EGP model and computer solution methodology can be applied to deal with these questions.

#### Model Description

The global model for the naval personnel assignment problem is illustrated in Figure 1. Proceeding from left to right, the sets of nodes in Figure 1 represent (1) naval personnel, (2) authorized billets, (3) activities or detachments, and (4) major claimants. The arcs (links) between node pairs indicate potential or existing paths. For example, the arcs between personnel and billet nodes indicate the eligibility of the person for the billet. Thus, according to Figure 1, person P1 may be assigned to billets B1, B3, or B4. The arcs between billet nodes and activity nodes indicate that each authorized billet is attached to a unique activity. Similarly, the activity/major claimant arcs indicate that each activity is attached to exactly one major claimant.

In the full version of the model, coefficients (not on diagram) are attached to the arcs. For example, attribute coefficients (such as permanent change of station cost, the man's desirability for the billet, and Navy's desirability to assign the man to the billet) are attached to the personnel/billet arcs. Similarly, an activity and/or major claimant attaches billet filling priority coefficients to the billet/activity arcs. Lower and upper bounds are associated with activity/major claimant arcs to indicate a range on the number of personnel assigned to the activity. Finally, the personnel nodes are given a supply of one indicating that each person is to be assigned to exactly one billet and each major claimant is given a demand range equal to its "fair share" manning level. The result is a global multiattribute transshipment network formulation of the naval personnel assignment problem.

A full version of this transshipment problem would involve approximately one million nodes. This substantially exceeds current solution capabilities which can handle at most 50,000 nodes. It should be noted, however, that alternative policies will drastically alter the number of nodes and arcs which would have to be considered in any problem related to this model. For example, depending on the projected rotation date (PRD) policy used, the number of men which are eligible for more than one billet may be less than 10,000 and the number of unfilled billets may be less than 15,000 or 20,000 billets. Consequently, the entire problem may be within current solution capability for some policies. However, as policies are allowed to vary, the problem can quickly exceed current solution capability. Thus, our goal is to derive an approximation to this model that abandons as little of its global nature as possible. This is accomplished by referring to a form of decomposition that aggregates critically interdependent activities in a single "bundle."

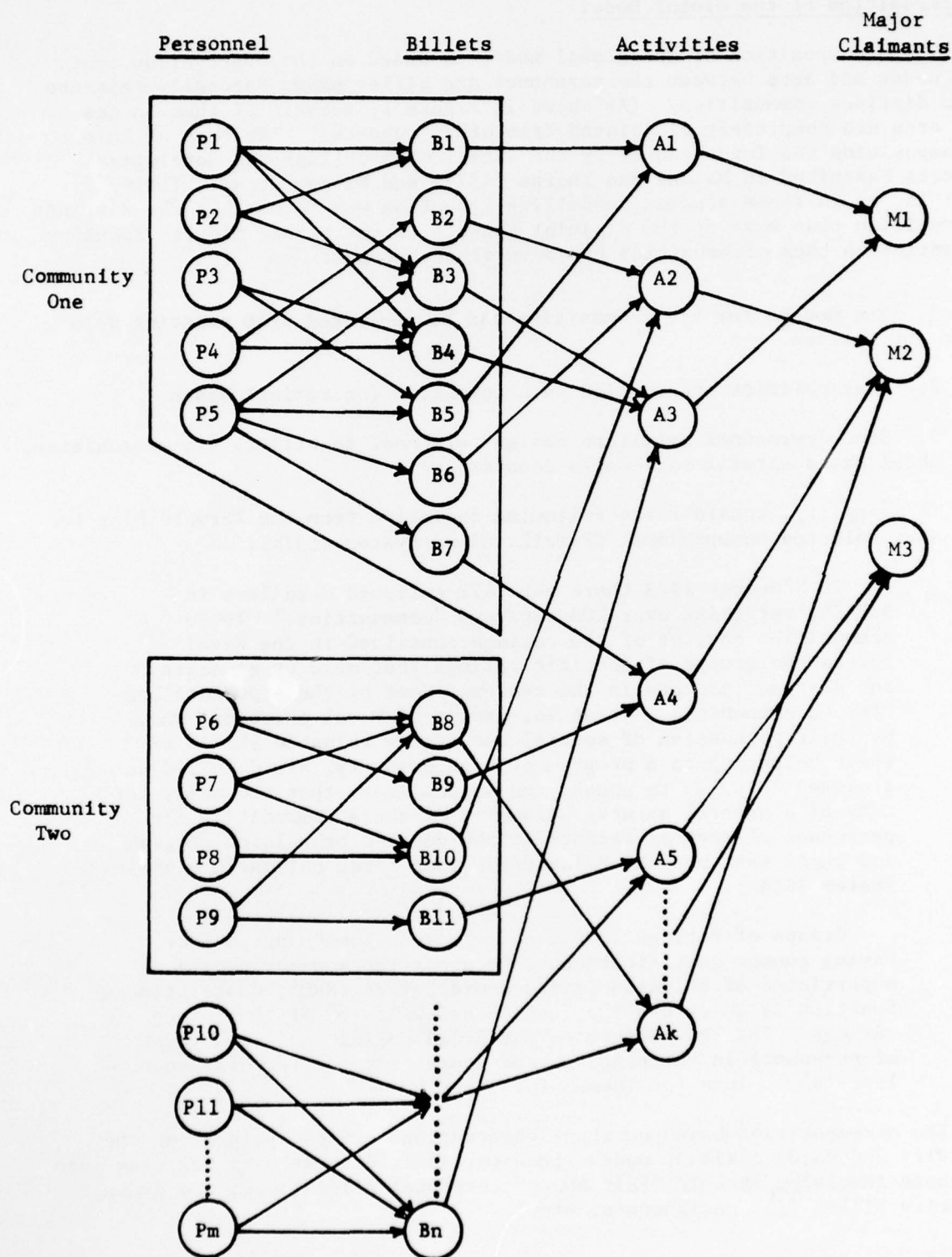


Figure 1. Global view of naval personnel assignment problem.



### Decomposition of the Global Model

The decomposition of the global model is based on the observation that the nodes and arcs between the personnel and billet nodes naturally separate into distinct communities. (As shown in Figure 1, subsets of these nodes and arcs are completely disjoint from other subsets.) The form of this decomposition was foreshadowed by the excellent modelling and development efforts described in Malone and Thorpe (1973) and Malone et al., (1974). That is, as in these studies, modelling attention was focused on the distinct communities that make up the disjoint subsets of the global model. Focusing attention on these communities has several advantages:

1. The models for the communities can be generated from existing data base information.
2. Most policies are applied on a community (or rating) basis.
3. Since personnel detailers assign personnel to billets via communities, the model has a natural real-world counterpart.

To exemplify, consider the following quotation from the Forward Plan for the Navy Enlisted Occupational Classification System (1974):

In November 1973 there were 172 enlisted detailers in BUPERS, servicing over 200 separate "communities." These communities consist of the ratings contained in the Naval forces and groups of specific ratings that make up separate and distinct pockets in the system. Some of these pockets are "fenced communities," that is, groups of Naval personnel who, by their possession of special and highly valuable skills or their belonging to a program of high priority, are detailed in a closed loop, so to speak, and seldom leave that community for duty of a general nature. Examples of these communities are personnel of several ratings in the nuclear propulsion program and those that man the Polaris/Poseidon fleet ballistic missile system (SSBN).

Groups of ratings that usually have a common purpose or having common qualifications come under the monitoring and supervision of enlisted rating coordinators (ERC), whose primary function is to ensure the "health and welfare" of that group of ratings. The ERCs determine the broad policies of employment of personnel in their groups and coordinate the formulation of long-range plans for them. (p. 64)

The decomposition based on these observations arises by ignoring the activity and major claimant nodes (thus partially disregarding policies such as those involving dynamic "fair share" personnel allocations, and dynamic priority billet fill assignments, etc.).

The network model that captures this type of decomposition is depicted in Figure 2. The data for the model is generated (see Malone et al., 1974) by looking through the enlisted master file and student avail file to find all assignment-eligible personnel in a particular community. The requisition files are used to create a list of all unfilled billets in this community. Eligible assignment arcs and multiattribute assignment coefficients are then determined. In the model included in Malone et al., (1974), the "cost" of not assigning a person was set at 999, thereby allowing incomplete assignments to be penalized at this figure for each individual unassigned. The method for handling this alternative was to create a dummy node to receive unassigned personnel, as shown in Figure 2.

The disadvantage of using the decomposition in the fashion depicted in Figure 2 (which is the state-of-the-art approach) is that the following highly important questions cannot be accommodated:

1. How should the personnel/billet attributes be used in assigning personnel to billets on a community basis?
2. Should filling the maximum number of unfilled billets with available personnel preempt all other considerations?
3. What should be the cost of not assigning a person?

Thus, we have treated the decomposition of the global model in a different way. The results of our efforts in this direction has led to the creation of an alternative (expanded) model that permits the foregoing policy questions to be accommodated. The details of this expansion and a complete discription of the new extended goal programming model are given in a subsequent section.

### Solution

Since the new model is substantially more sophisticated than those previously considered, it is unfortunately not amenable to solution by the standard network solution algorithms. In fact, the new model is technically an integer linear programming problem--or more specifically, an integer constrained transportation problem (Klingman & Russell, 1975). Thus, its solution requires a major advance in solution methodology and computer implementation techniques. To this end, we have devised an approximation method for obtaining good integer solutions based on refinements and specializations of subgradient optimization techniques (see Glover, 1975; Held, Wolfe, & Crowder, 1974). The major contribution of our development is to enable us to utilize the efficient network code developed by Barr et al., (1974) to obtain good approximation solutions. The foundations of this development are described in a later section.

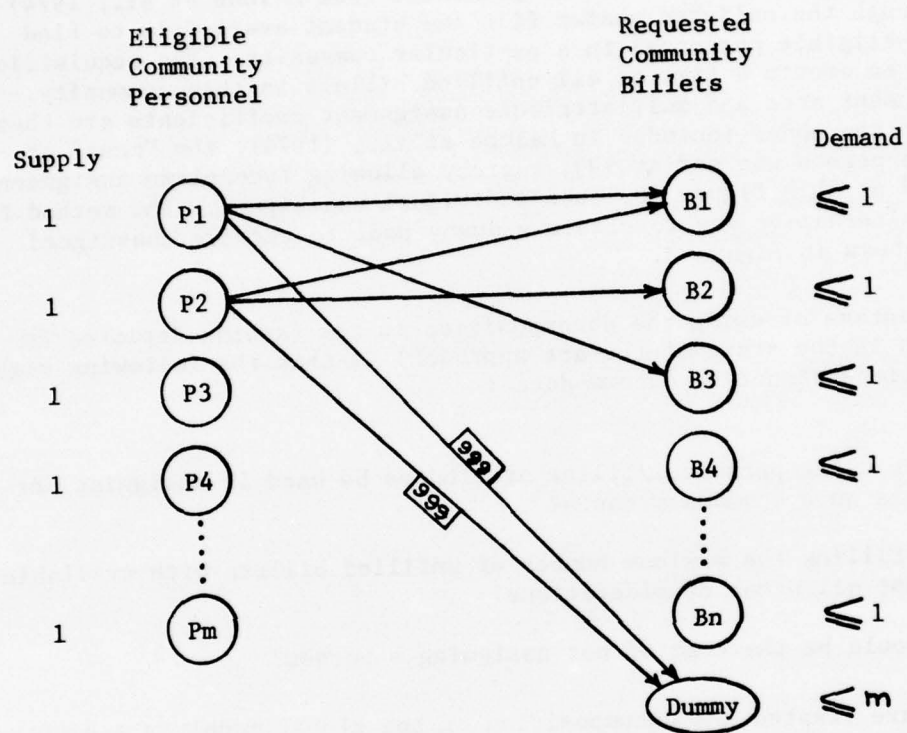


Figure 2. Decomposition model for a typical community.



## EXTENDED GOAL PROGRAMMING MODEL

### Approaches to Manpower Modelling

The fundamental policy evaluation testing model utilizes a combination of two strategic concepts that are new both to manpower planning and to mathematical programming generally. These concepts, dynamic sensitivity analysis and extended goal programming, provide a methodology capable of determining accurate trade-offs between competing goals in complex decision-making settings.

To get a clearer picture of these concepts and the manner in which they have been utilized in creating a policy evaluation model for NAVPERSRANDCEN, it is appropriate to sketch briefly the analytic techniques that would ordinarily be applied in such a setting and to indicate their shortcomings. There are three primary goals that the manpower assignment model seeks to accommodate. These arise by characterizing the cost of assigning a man to a billet, the utility of such an assignment to the Navy, and the desirability of the assignment to the man himself. These three factors can be summarized by specially constructed cost, utility, and desirability functions. By these constructions, all three functions may be viewed as "cost" functions in the sense that higher values correspond to less desirable alternatives. Consequently, the overall objective of assigning personnel to billets may be viewed as that of minimizing each of these three types of costs (hence, optimizing true cost, utility to the Navy, and desirability to the assigned personnel).

Unfortunately, of course, it is impossible to obtain an assignment that is simultaneously optimal for all three goals imbedded in the overall objective. As a result, it becomes important to identify a way to respond effectively to the competing characterization of combined optimality.

One approach (one that is intuitively natural as a limited first approximation) is to specify a weight for each goal that reflects its significance relative to the other goals. Thereupon, a linear combination based on these weights is applied to the "cost" functions associated with these goals. The result is used as a single "composite" objective function to be minimized by an appropriate optimization method. This, in fact, is the approach used by Malone and Thorpe (1973) in their preliminary study. Malone and Thorpe noted the inadequacy of the approach and suggested that a sophisticated "goal programming" type of approach should be considered. However, after carefully considering the use of the standard goal programming approach, we recently discovered that it had a serious shortcoming which is linked to the same fundamental principle that renders Malone and Thorpe's "composite" objective function approach inadequate. The principle is easy to state: there is no way to know a priori how to weight two different objective functions or "goal constraints" so that they will have equal significance--or so that they will have a relative significance specified by a particular ratio. (In more general terms, there is no way to know how to weight a collection of constraints in order to give the members a desired relative significance.) This difficulty may arise from different relative amounts of the same unit of measurement or from different units of measurement.

Another type of approach that is also relatively popular and that seems intuitively reasonable is "surface" optimization (Hatch, Nauta, & Pierce, 1972). This approach has some appeal because of its simplicity and its ease of implementation. However, the solutions obtained by this approach are subject to even more serious defects than those obtained by standard goal programming. Rather than attempt to weight the goals according to their relative importance, surface optimization arranges the goals in an absolute hierarchy. Thus, instead of seeking solutions that are optimal in terms of preferred tradeoffs among goals, this strategy seeks a solution that is optimal in terms of the top ranked goal, and then, subject to this, in terms of the next ranked goal, and so on. The shortcoming of the approach is that it follows a "devil take the hindmost" philosophy. That is, if optimal solutions to the first ranked objective are extremely poor in terms of the second ranked objective, the second objective is left to suffer the consequences, and no attempt is made to find a satisfactory alternative that may be good in terms of both objectives. Further, by the time the first two objectives have been accommodated in this fashion, there is little chance that a third objective will be accommodated to any significant extent. Thus, in situations where multiple goals are meaningful and deserve more than token consideration--as in the manpower assignment context--the surface optimization approach has conspicuous shortcomings. On the other hand, if the goals should be ranked in an absolute hierarchical fashion, this can be achieved even by standard goal programming. That is, surface optimization is mathematically a special case of standard goal programming, and the limitations of this more general approach have already been noted.

In order to overcome the shortcomings of these standard techniques as applied to manpower planning problems, it is first necessary to have a meaningful way of characterizing the relative significance of different objectives. To say that one objective is "twice as important" as another is not sufficiently precise in a mathematical sense. There must be a context for the "twice as important" evaluation. This context is lacking in previous manpower planning applications. Further, beyond context, there must be a way of modelling the relative significance of competing goals to respond to constraints imposed by the system under consideration. These are the features that are captured in the model developed for NAVPERSRANDCEN policy evaluation of personnel assignment.

#### Model Description

To describe the EGP model, we must leave the realm of analogy and introduce specific mathematical notation. Accordingly let

$x$  = the vector of decision variables that characterize assignments of personnel to billets,

$c$  = the vector of coefficients for the cost function ( $cx$ ),

$u$  = the vector of coefficients for the utility function ( $ux$ ), and

$d$  = the vector of coefficients for the desirability function ( $dx$ ).



Further, subject to the basic assignment network of Malone and Thorpe (1973) and Malone et al. (1974), let

$c^* =$  the minimum value of  $c_x$ ,

$u^* =$  the minimum value of  $u_x$ , and

$d^* =$  the minimum value of  $d_x$ .

Each of  $c^*$ ,  $u^*$ , and  $d^*$  can be obtained independently of the others by an appropriate network solution method. It follows from these definitions that

$$c_x \geq c^*, u_x \geq u^*, \text{ and } d_x \geq d^*$$

must hold for all admissible assignments of personnel to billets. In an utopian situation, one would seek an assignment such that  $c_x = c^*$ ,  $u_x = u^*$ , and  $d_x = d^*$ . Since this cannot happen, we may consider an "extended goal" of finding an optimal solution in which the percentage deviation of each function from its minimum is constrained to the same value. It can be shown that this is equivalent to solving a problem of the form:

Minimize  $c^1 x$

subject to:

the original network constraints  
and  $c^1 x = u^1 x = d^1 x$   
where  $c^1 = c/c^*$ ,  $u^1 = u/u^*$ ,  $d^1 = d/d^*$ .

To reduce round-off error it may be useful to divide  $c^*$ ,  $d^*$ , and  $u^*$  by the minimum of these three values, so that the new  $\min = 1$  before defining  $c^1$ ,  $d^1$ , and  $u^1$ .

However, the equality constraints of the foregoing system, after preliminary analysis, still turn out to be unduly restrictive. Superior solutions can be obtained by utilizing the special inequality system:

$$u^1 x - 1 \leq c^1 x - 1$$

$$d^1 x - 1 \leq c^1 x - 1.$$

The "-1's" in these inequality constraints would ordinarily cancel each other and be dropped. However, they are retained to make it possible to deal with situations in which the relative significance of deviating from a minimum can be expressed by a ratio. That is, the "percentage deviation from minimum" concept imbedded in the foregoing system gives a context relative to which weighting schemes can become meaningful. Specifically, if each percentage unit deviation of  $d_x$  from its minimum is three times as undesirable as each percentage unit of  $c_x$  from its minimum, then the appropriate representation is

$$3(d^1 x - 1) \leq c^1 x - 1$$

or

$$3d^1 x - c^1 x \leq 2.$$

By minimizing the  $c^1x$ , the foregoing inequalities bound all deviations from minima with a value that is as tight as possible. More precisely, as a result of the minimization, the inequalities control the worst percentage deviation, while allowing other deviations actually to be better. It is this form of combined objective function and auxiliary inequality system that we call "extended" goal programming. By failing to incorporate functional interdependencies into the model structure, it is not surprising that ordinary goal programming and surface optimization are incapable of providing solutions in which the relative impact of the various functions can be controlled or assessed. Moreover, the fact that extended goal programming incorporates one of the functions into the objective to be minimized, and uses this function to transmit the appropriate interrelationships to the other functions, overcomes the necessity to know in advance how the functions are affected by the structure of the feasible solution space. The ultimate interactions of all constraining relationships are established via the minimization process, rather than requiring a priori information.

A key to the potency of the approach, which enables it to overcome the shortcomings of other approaches, is not merely its ability to supply a meaningful context for relative deviations, but more importantly derives from the fact that the auxiliary constraints are expressed as relationships between functions. In ordinary goal programming and surface optimization, the constraints do not utilize or impose functional interrelationships, but treat each function independently of the others.

## SOLUTION APPROACH

### Constraint Accommodation

To handle the extended goal programming model with state-of-the-art solution technology, it is necessary to determine a way to accommodate the additional constraints. This is highly important, because the additional constraints destroy the pure network structure of the original problem and make it intractable for ordinary techniques. Dynamic sensitivity analysis enters as a way to overcome this limitation. This technique attempts to ascertain the optimal objective function composition that absorbs these constraints and leaves the pure network structure intact.

Stated in other terms, dynamic sensitivity analysis is a vehicle for creating a composite objective function that "corresponds to" the extended goal programming constraints. Thus, in the same way that the extended goal programming model structure overcomes a fundamental flaw in the standard goal programming approach, so does its associated composite objective function overcome corresponding flaws in the standard composite objective function.

Necessarily, dynamic sensitivity analysis is a staged interactive process. Just as extended goal programming captures the ultimate interactions between competing goals by means of the minimization process itself, so does dynamic sensitivity analysis capture an appropriate composition for the objective function by reference to this same solution process. Thus, once again, the recourse to blind a priori parameter settings is avoided.

In fact, dynamic sensitivity analysis not only determines the suitable composite objective function but also provides a means for analyzing alternative assignment criteria. Such criteria can include alternative types of preemptive policy-setting parameters, making it possible to evaluate surface optimization and other preemptive strategies as a special case. Thus, dynamic sensitivity analysis constitutes a means for supplementing the extended goal programming model with an evaluative component. Given the inability of existing solution methods to handle a problem in which the extended goal programming constraints are incorporated directly into the model structure, dynamic sensitivity analysis further provides a means of creating a composite objective function with desirable characteristics. This makes it possible to approximate the use of extended goal programming constraints and thereby to take advantage of existing solution capabilities (as augmented to include the techniques for implementing dynamic sensitivity analysis itself).

### Dynamic Sensitivity Analysis

To create a composite objective function that exhibits the unique properties just discussed, dynamic sensitivity analysis utilizes a form of generalized Lagrangean relaxation. The extended goal programming model and solution concepts form the heart of the approach. In overview, the analysis may be viewed as a procedure for progressively amending the initial objective function  $c^1x$  to capture the extended goal programming interrelationships as nearly as possible. This progressive amendment occurs by assigning Lagrangean multipliers to the inequalities of the extended goal programming



system and systematically adjusting these multipliers until a saddle point solution is reached. (The "saddle point" terminology is relevant because the generalized Lagrangean relaxation corresponds in this setting to seeking an optimal solution to a dual linear program.) The multipliers that are finally determined in this procedure are then translated into weights for each of the separate functions  $cx$ ,  $ux$ , and  $dx$ . The significance of this translation derives from the fact that these weights can subsequently be applied in solving other problems by a composite objective function approach.

By its intimate connections to extended goal programming and its iterative nature, this phase of the dynamic sensitivity analysis constitutes an entirely new tool for determining and optimizing a composite objective function. Further, by its use of Lagrangean relaxation, the analysis incorporates a technique whose power and efficiency are already established in other settings. This is extremely important because the use of efficient procedures for optimizing personnel assignment and policy evaluation models has been noted as indispensable (Borgen & Thorpe, 1967; Malone et al., 1974). Accordingly, one of the primary goals of the current effort is to identify an effective way of implementing the steps of the Lagrangean relaxation approach in the specialized network setting of the current models. Techniques for accomplishing this involve tailoring the Lagrangean techniques to the data representations and algorithmic processes of the underlying network optimization method. Additionally, this specialization involves a major original research component for ascertaining efficiency and valid step size and search procedures. These implementation facets of the Lagrangean concept have been incompletely addressed in the literature. As stated in Held, Wolfe, and Crowder (1974), "choice of step size is an area which is imperfectly understood" (p. 68).

To make the foregoing notions more precise, the complete form of the extended goal programming model will be expressed in the following notation:

$$\text{Minimize } c^1x \quad (1)$$

subject to:

$$Ax = b \quad (2)$$

$$\alpha u^1x - c^1x \leq \alpha - 1 \quad (3)$$

$$\beta d^1x - c^1x \leq \beta - 1 \quad (4)$$

$$x \geq 0 \text{ and integer} \quad (5)$$

The vectors  $x$ ,  $c^1$ ,  $u^1$ , and  $d^1$  are previously defined, and the constraints (2) and (5) represent the constraints of the network decomposition system (shown in Figure 2) independent of the extended goal programming problem. (For example, the value  $c^*$  that determines the vector  $c^1$  from the equation  $c^1 = c/c^*$  is given by  $c^* = \text{Min } cx$  subject to (2) and (5).)

The constants  $\alpha$  and  $\beta$  are the relative weights attached to percentage deviations from the targeted optima. Thus, by the interpretation illustrated earlier, these constants correspond to viewing each percentage deviation of

cx from  $c^*$  as having " $1/\alpha$  times" the undesirability of each percentage deviation of  $u_x$  from  $u^*$ , and " $1/\beta$  times" the undesirability of each percentage deviation of  $d_x$  from  $d^*$ . (If  $\alpha = \beta = 1$ , then all deviations have the same level of undesirability.) The constants  $\alpha$  and  $\beta$  are established in advance according to the relative significance attached to each of the "percentage deviation" goals by those responsible for setting policy. (In addition, the influence of attaching particular degrees of significance to each of the goals can be assessed by treating these constants or parameters internal to the dynamic sensitivity analysis procedure. This represents one of the important side benefits of the model.)

Now to accommodate (3) and (4) indirectly by generalized Lagrangean techniques (in the first phase of dynamic sensitivity analysis), nonnegative "multipliers"  $w_1$  and  $w_2$  are associated respectively with these two constraints to create the new objective function.

$$\text{Minimize} \quad c^1x + w_1(\alpha u^1x - c^1x) + w_2(\beta d^1x - c^1x) \quad (1')$$

subject to:

$$Ax = b \quad (2)$$

$$x \geq 0 \text{ and integer.} \quad (5)$$

The new problem is a pure network problem (since its only constraints are (2) and (5)), and hence is susceptible to exploitation by efficient network procedures. Problem (1'), (2), (5) is not equivalent to the original problem (1) - (5) since it admits a wider range of possible solutions. However, for "best" values of the multipliers  $w_1$  and  $w_2$ , the new problem approximates the original problem and, under fortunate circumstances, an optimal solution to the new problem can turn out to be optimal for the original.

The ability of (1'), (2), (5) to serve as an "approximating problem" under appropriate values of the multipliers constitutes its primary significance, together with the fact that the region on which (1') is defined is concave for nonnegative values of  $w_1$  and  $w_2$ . This concavity implies unimodality and thus further implies that the best values of  $w_1$  and  $w_2$  (which achieve a Max-Min objective) can be determined by a subgradient search procedure (Glover, 1975; Held et al., 1974).

Specializing such a procedure to the network structure and conducting tests to determine the most effective subgradient, search procedures, and step sizes constitutes another key effort of our investigation. The trade-offs in maximizing efficiency and minimizing additional core memory have been given careful scrutiny. The chief objective of this phase of the overall dynamic sensitivity analysis is to determine subgradient search techniques to merge with the existing network computer code (Barr et al., 1974) in order to determine best values of the constraint multipliers in an efficient manner. Because the network method must be successively iterated to accomplish this determination, the precise interactions between the search techniques and network code are of crucial importance.

Upon obtaining the sought-after values of  $w_1$  and  $w_2$ , which we denote  $w_1^*$  and  $w_2^*$  respectively, we utilize this information to obtain an appropriately weighted composite objective function in the original problem. As previously stressed, there is no a priori way to determine a meaningful composite objective function, but the foregoing procedure, beginning with the extended goal programming model and then applying Lagrangean relaxation, provides such an objective function in an adaptive manner. In fact, it is completely unnecessary at the conclusion of the process to reoptimize relative to the composite objective function thus derived, because optimization relative to this objective has already implicitly been carried out. Nevertheless, the recovery of such an objective function, and the weights on which it is based, can be useful for subsequent applications and for interpreting the significance of the components of the composite objective function. Thus, specifically, the formula for the composite objective function is:

$$(y_1c + y_2u + y_3d)x$$

$$\text{where } y_1 = (1 - w_1^* - w_2^*)/c^*, y_2 = \alpha w_1^*/u^*, y_3 = \beta w_2^*/d^*.$$

By means of this formula the remarkable effect of utilizing the extended goal programming model becomes apparent. It is entirely possible for  $y_1$  to receive a negative value in this assignment, which seems thoroughly counter-intuitive until the derivations are traced back to the original extended goal programming model.

The possibility of a negative assignment reflects the fact that any meaningful definition of "composite optimality" must rely on the interdependency of the functions  $cx$ ,  $ux$ , and  $dx$ . Clearly, it would be totally impossible to infer the consequences of this interdependency in terms of the above expressions given  $y_1$ ,  $y_2$ , and  $y_3$  were it not for the extended goal programming framework. Indeed, it would be particularly desirable to trace the consequences of a direct application of this model, instead of an indirect application through the intermediate approximating problem (1'), (2), (5).

#### Preemptive Policy Evaluation

The second phase of the dynamic sensitivity analysis provides a means to capture and evaluate the effects of a preemptive policy that assigns the maximum number of personnel to billets. Previous studies have not been able to isolate the influence of such a preemptive policy or to determine the trade-offs between maximum personnel assignments and assignments that maximize other measures such as utility and desirability. In fact, the model structure we propose automatically yields optimal assignments via the extended goal programming analysis just discussed, given the preemptive policy. Moreover, the completed second phase of dynamic sensitivity analysis yields a characterization of the cost of preemptive and "near" preemptive policy assignments.

To accomplish this, the personnel assignment model is first solved using the preemptive objective to identify the maximum number of personnel that can be assigned to billets. We let  $m^*$  denote this maximum number and let  $m$  denote the total number of personnel available to be assigned. Thus,  $m - m^*$  is the number of personnel that cannot be assigned. If  $n$  represents the total number of billets under consideration, then  $n - m^*$  is the number of billets not currently filled.



Using this information, the assignment model is modified in the following manner. A source node (supplementary) and a sink node (supplementary) are incorporated into the model with a supply of  $n - \gamma m^*$  and a demand of  $m - \gamma m^*$ , respectively, where  $\gamma$  is the minimum percentage of maximum fill required. (Each personnel node is given a unit supply and each billet node is given a unit demand.) Zero-cost arcs are created leading from each of the personnel nodes to the sink and, also, leading from the source to each of the billet nodes. In addition, a special arc is created from the source to the sink which has a cost  $-KOST$ , where  $KOST$  is a positive number. (Thus the cost of  $-KOST$  may be viewed as a profit.) Costs from the composite objective function previously identified are attached to the arcs leading from the personnel nodes. Figure 3 provides a diagram of this model structure. This model structure assures that a preemptive assignment of a maximum number of personnel to billets will automatically be achieved if  $\gamma = 100\%$ . Given the preemptive policy, an optimal assignment is automatically obtained relative to the other criteria of cost, utility, and desirability. More precisely, each of the modelling and solution concepts underlying extended goal programming and dynamic sensitivity analysis--as previously described--can be directly applied to the network of Figure 3. This means that the supplementary constraints and the composite objective function can be utilized to reflect the special circumstances of the preemptive policy structure. There is no need to generate these constraints and objective functions for a different structure and then attempt (in a less than adequate manner) to apply them to the preemptive situation.

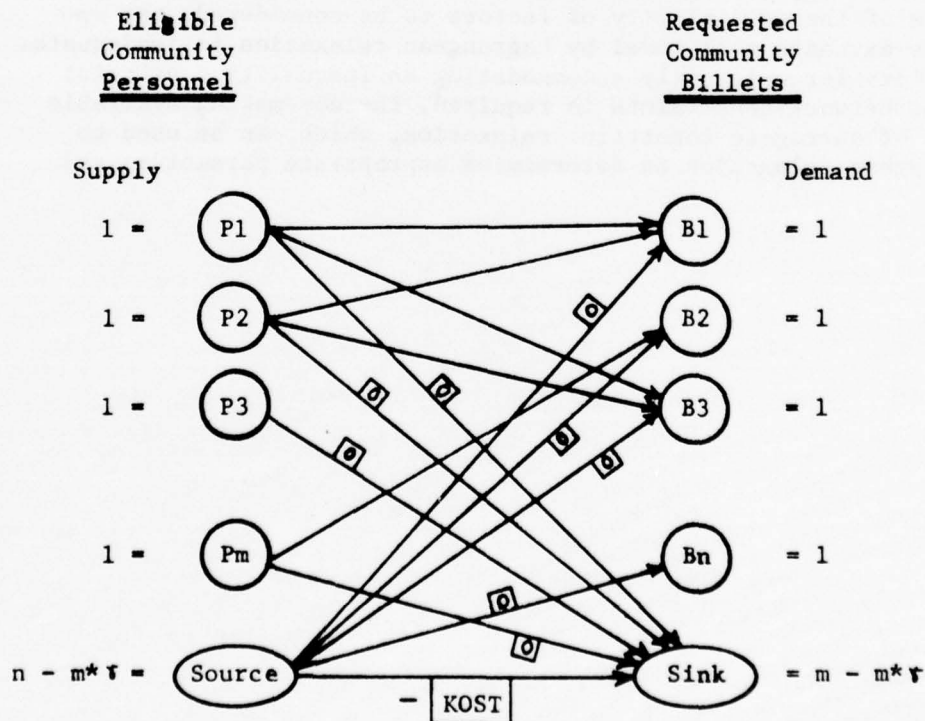


Figure 3. Pre-emptive policy evaluation.

An additional feature of this model provides the most important aspect of its sensitivity analysis. This occurs by varying the value of KOST and  $\gamma$  and adjusting the parameters of the extended goal programming model (e.g., via Lagrangean relaxation) to their new optimal values. In this manner, the full operation of dynamic sensitivity analysis is achieved. This analysis gives rise to graphical representations of the trade-offs between the cost, utility, and desirability criteria relative to varying levels of importance attached to the total number of men assigned to billets. Specifically, as changed values of KOST and  $\gamma$  yield different solutions and trade-offs, a measure is obtained of the relative cost of achieving various percentages of the maximum assignment of personnel to billets. In this way, the implications of preemptive and "near" preemptive policies can be assessed. The graphical portrayal of the interrelationships of the relevant factors provides a framework for viewing the consequences of alternative policy considerations.

An attractive feature of this model approach is that it lays the foundation for accommodating additional considerations of personnel assignment that have previously been beyond the reach of analysis. In particular, the cost, utility, and desirability criteria are themselves aggregates of other underlying factors. A major potential criticism of the model--and of essentially all models of this same fundamental character--is that there must always remain some doubt about the meaningfulness of its solutions as long as no way exists to analyze the relative contributions of these basic constituents. Now, however, the extended goal programming framework opens the door to such analysis. Because of the multiplicity of factors to be considered, the approximation to the extended goal model by Lagrangean relaxation is inadequate. Instead, a capability for explicitly accommodating an inequality constraint in addition to the network constraints is required, thereby making available the powerful tool of surrogate constraint relaxation, which can be used to augment the Lagrangean relaxation in determining appropriate parameters and solution values.



## PROTOTYPE COMPUTER-ASSISTED POLICY EVALUATION SYSTEM

### Computer Implementation

The essential practical considerations underlying computer implementation have remained at the forefront in the design of the models and solution approaches described in the preceding sections. The purpose of this section is to detail these considerations and the structure of our computer implementation approach.

An effective policy evaluation tool must of course rest heavily on the manner in which analytical models and solution techniques are incorporated into usable computer programs. The programming system we have developed is descriptively called the Prototype Computer-Assisted Policy Evaluation (CAPE) System. In designing CAPE we studied the functions of the Computer-Assisted Detailing and Assignment (CADA) System (Malone et al., 1974) and met with Malone in order to ascertain the strengths and shortcomings of the CADA implementation. After studying CADA we chose to develop an entirely new system with the exception of the front end routines. We found these routines to be sufficiently flexible in their format and input configurations that we were able to modify this portion of CADA in a convenient fashion to provide the inputs to our more general model and solution routines. These latter routines, which provide the body of our new system, are described subsequently. We are indebted to Malone for assisting us in the evaluation and for providing us with a copy of CADA and data for validating the CAPE System.

Although the CAPE System presently encompasses only the Disbursing Clerk (DK), Aviation Maintenance Administrationman (AZ), and Hospital Corpsman (HM) ratings, it is generalizable to the other enlisted ratings.

### Comparison of CAPE and CADA

The CAPE System provides the first capability for enforcing desired proportionalities between conflicting objectives in manpower planning applications. The extended goal programming model imposes functional interrelationships on the cost, utility, and desirability functions and thus is able to characterize in a meaningful context the relative significance of these competing goals. The CADA System relies on a single composite objective function constructed from a linear combination based on weights for each goal which must be specified a priori. The CAPE System, on the other hand, avoids the inadequacies of this approach by treating the auxiliary constraints of the extended goal programming model explicitly and establishing the ultimate interactions of all constraining relationships via the minimization process.

Furthermore, the CAPE System adds a means of evaluating the effects of policies dealing with the number of personnel assigned to billets. It is now possible to isolate the influence of the preemptive policy of assigning the maximum number of personnel to billets and to determine the trade-offs between maximum personnel assignments and assignments that maximize other measures such as utility and desirability. Thus, the CAPE System is capable of obtaining a measure of the relative cost of achieving various percentages of the maximum assignment of personnel to billets. The CADA System, on the contrary, has no facility for enforcing a given level of personnel assignments.

## CAPE System Overview

The CAPE System (see Figure 4) is comprised of three major sections: the input interface (CAPE 201 and CAPE 202), the model construction phase (CAPE 203 and CAPE 204), and the optimization phase (CAPE 205).

### Input Interface Phase

In the input phase, CAPE 201 extracts from the student avail file a set of candidates eligible for assignment. Provision has been added for user specification of the length of time (number of months) prior to an individual's projected rotation date (PRD) that he will be considered for transfer. CAPE 202 produces a list of eligible requisitions from the LANT, PAC, and BUPERS Requisition Files. CAPE 201 and CAPE 202 were taken directly from CADA 201 and CADA 202, respectively.

### Model Construction Phase

CAPE 203 examines the lists of qualified candidates and requisitions produced by CAPE 201 and CAPE 202, ascertains man/billet eligibilities, and computes the components of the cost, utility, and desirability functions. CAPE 203 is the same as the CADA 203 routine. From this set of man/billet combinations, CAPE 204 constructs the final form of the model as depicted in Figure 3. In this procedure the cost, utility, and desirability functions are standardized to fall in the range 1-900,  $\gamma$  is set to 0 (so that  $m^* = 0$ ), and KOST is set equal to 999. In addition, a master source node, a master sink node, and the appropriate arcs are added to produce a circularized network. The appendix presents the program documentation for CAPE 204.

### Optimization Phase

The optimization phase, CAPE 205, consists of a specialization of the computationally efficient network code (Barr et al., 1974) integrated with a user-oriented front end for defining the desired policies to be employed. The interface allows the user to specify what proportional relationships are desired between the utility, desirability, and cost functions (i.e., the  $\alpha$  and  $\beta$  values) and the type of policy to use with regard to maximizing the number of billets filled (e.g., maximize the number of billets regardless of other considerations, fill at least a user specified percentage  $\gamma$  of the maximum number possible taking account of the utility, desirability, and cost aspects where each unfilled billet is given a user specified "KOST," etc.). Default values for unspecified proportionality weights are unity. An unspecified "percentage maximum desired fill" is assumed equal to 100%, and an unspecified cost for an unfilled billet is assumed to be 999.

CAPE 205 then solves a maximum flow problem to determine  $m^*$ . The result is used to construct three assignment-transportation problems (similar to Figure 3), which are then to be solved in succession to yield  $c^*$ ,  $d^*$ , and  $u^*$ . The solutions to these latter problems are then integrated with the user-specified policies to construct the appropriate extended goal programming (EGP) model. The EGP model, as approximated in the manner previously described, is then solved to determine a set of nomination (person/billet) combinations which optimizes the dynamically created Lagrangean approximation to the EGP model subject to the specified assignment policies.

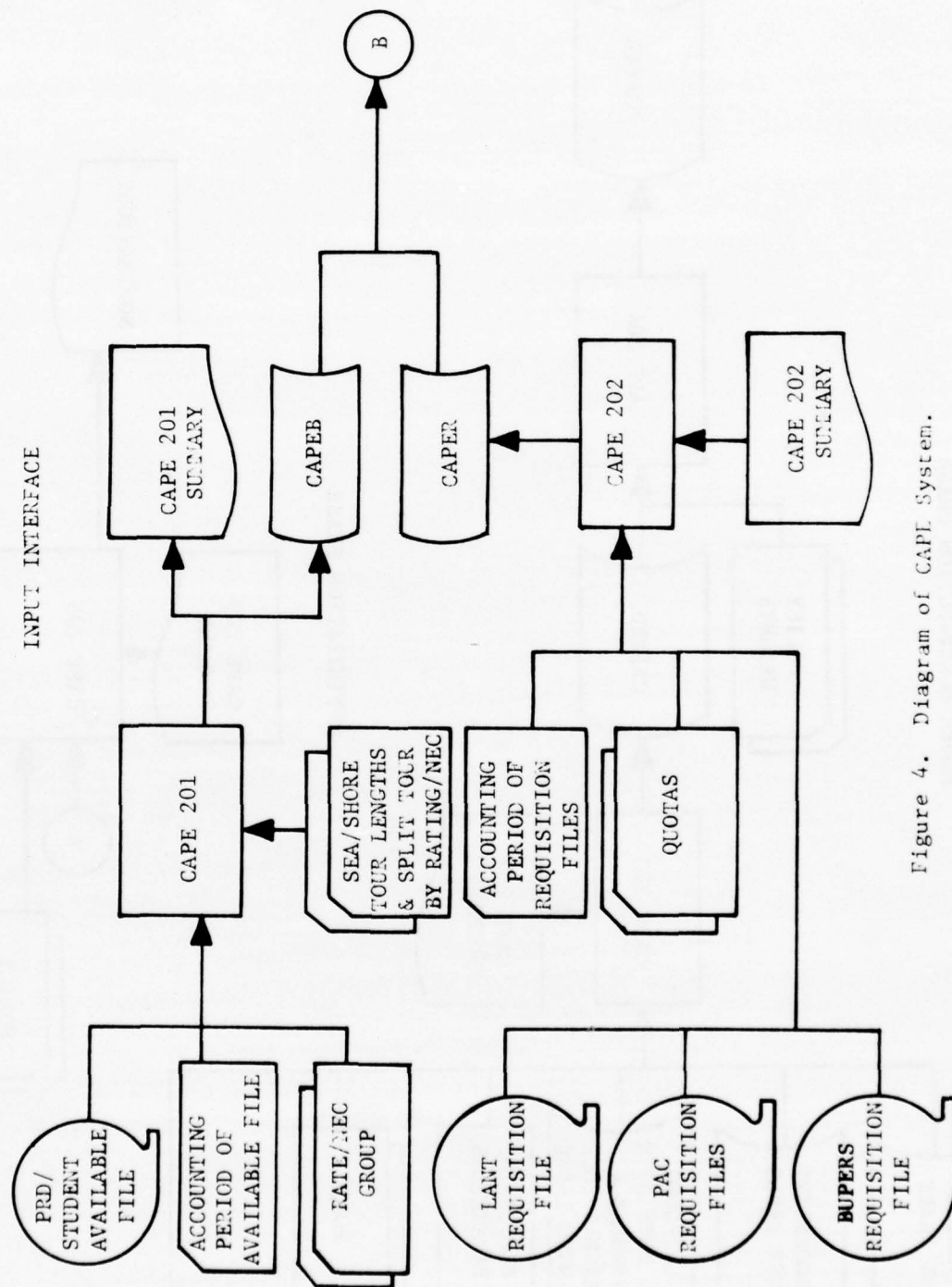


Figure 4. Diagram of CAPE System.

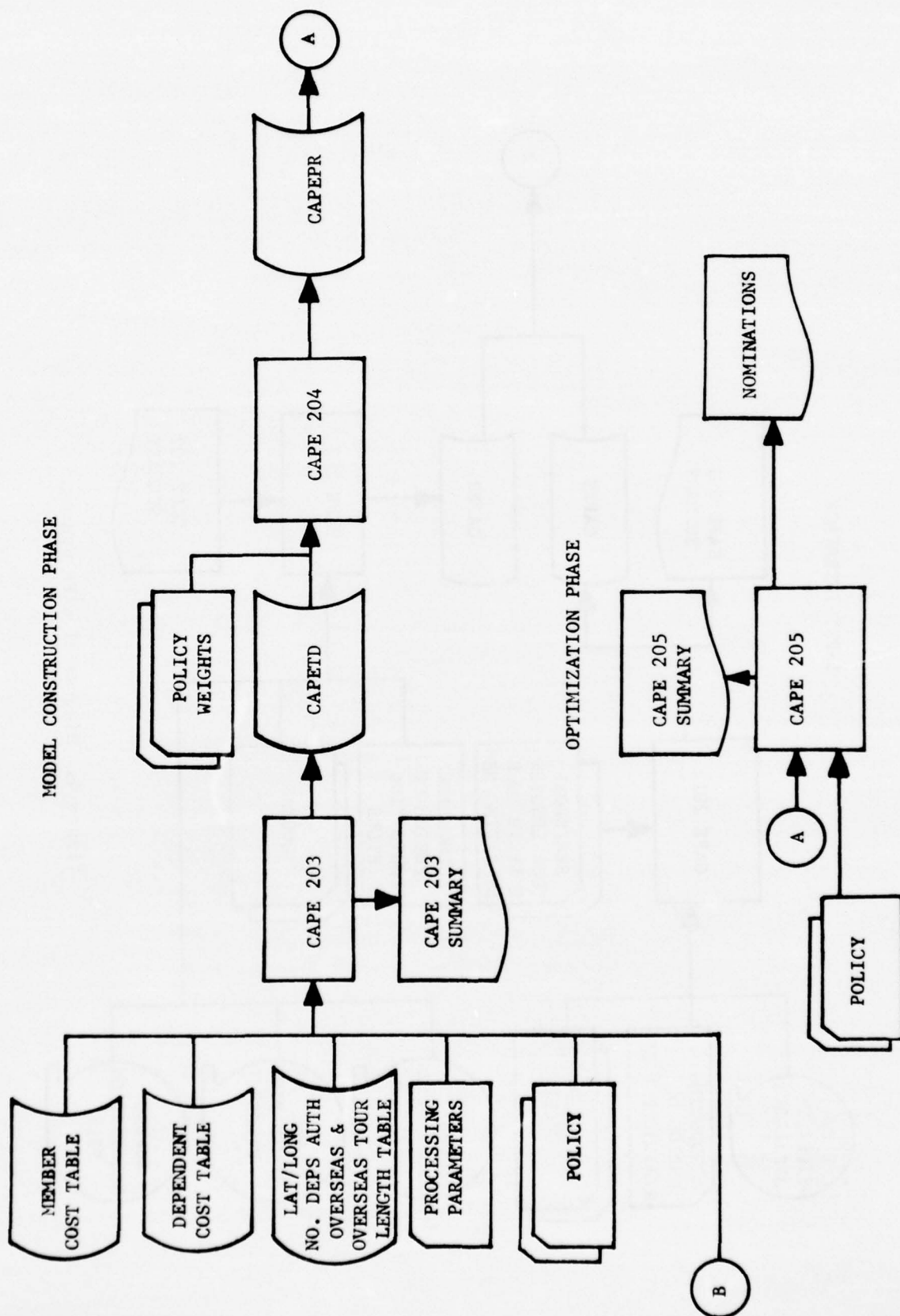


Figure 4. (Continued)



The optimization phase input consists of the output from the model construction phase and the user-specified policies. The output is (1) a list of person/billet nominations, (2) the maximum number of personnel that can be assigned, (3) the actual number of personnel assigned, (4) the user specified policies in the policy input and model construction phase, (5) the optimal value of the utility, desirability, and cost functions, (6) the actual value of the utility, desirability, and cost functions for the specified nominations, and (7) the optimal weightings of these functions in the composite objective function (i.e., the  $y_1, y_2, y_3$  values). Program documentation for CAPE 205 is included in the appendix.

#### Derivation of Lagrangean Step Size and Search Procedures

Extensive research and testing was conducted to find efficient step size and search procedures for implementing the Lagrangean techniques utilized in the solution of the EGP model. The basic strategy can be summarized as follows:

The problem to be solved is

$$\text{Minimize} \quad c^1 x + w_1(\alpha^1 x - c^1 x) + w_2(\beta^1 x - c^1 x) \quad (1')$$

$$\text{subject to:} \quad Ax = b \quad (2)$$

$$x \geq 0 \text{ and integer} \quad (5)$$

Let  $x^i$  be the optimal solution at step  $i$  and define

$$z^i = c^1 x^i + w_1(\alpha^1 x^i - c^1 x^i) + w_2(\beta^1 x^i - c^1 x^i)$$

$$s_1^i = \alpha - 1 - \alpha^1 x^i + c^1 x^i$$

$$s_2^i = \beta - 1 - \beta^1 x^i + c^1 x^i$$

0. Set  $w_1^0 = w_2^0 = 0$ ,  $\pi = \pi_0$ . Solve (1') giving  $x^0$ .

If  $s_1^0 \geq 0$  and  $s_2^0 \geq 0$ , stop. Otherwise, set  $i = 0$ ,  $k = 1$ , and

go to Step 1.

1. Set  $w_1^{i+1} = w_1^i - s_1^i \pi$

$$w_2^{i+1} = w_2^i - s_2^i \pi.$$

If  $w_j^{i+1} \leq 0$ ,  $j = 1, 2$ , set  $w_j^{i+1} = 0$ . Solve (1') yielding  $x^{i+1}$  and  $z^{i+1}$ .

2. If  $z^{i+1} \geq z^i$ , go to Step 3. Otherwise increment  $k$  by 1. Then if  $k > K$ , stop. Else, set  $\pi = \pi/2$  and go to Step 1.
3. Stop if  $z^{i+1} - z^i \leq \epsilon_1$  or  $|s_1^{i+1}| \leq \epsilon_2$  and  $|s_2^{i+1}| \leq \epsilon_2$ .  
Otherwise, increment  $i$  by 1, set  $k = 1$ , and go to Step 1.

When the algorithm terminates, it is possible that  $s_1^i < 0$  or  $s_2^i < 0$ , i.e., that the constraints (3) and (4) of the EGP model are not satisfied. In this event, a search is performed to locate the "best" solution for which  $s_1^i > 0$  and  $s_2^i > 0$ . This procedure is simply one of increasing the  $w_j$  just to the point where the  $s_j$  changes sign (from negative to positive). These values are then taken as the optimal values  $w_1^*$  and  $w_2^*$  used in computing  $y_1, y_2$ , and  $y_3$ .

Several algorithmic variations were also tested. For example, at Step 3, if  $s_j^{i+1}$  and  $s_j^i$  are of opposite signs, a search can be performed to find a step size yielding  $s_j^{i+1} = 0$ . Numerous strategies for adjusting  $\pi$  to find the zero were tried. However, in all cases it was difficult to find  $\pi$  giving  $s_j^{i+1}$  close to zero, and thus there was little benefit actually realized from this search.

The performance of the above procedure was tested for several different values of  $\epsilon_1$  and  $\epsilon_2$ , the initial step size  $\pi_0$ , and the maximum number  $K$  of iterations in Step 2. The efficiency of the procedure was very sensitive to the value of  $\epsilon_1$  and  $K$  but rather insensitive to the values of  $\epsilon_2$  and  $\pi_0$ . The value of  $\epsilon_2$  had little effect because the  $s_j$  frequently never got very close to zero regardless of the values of the  $w_j$ . Unless  $\pi_0$  was initially chosen much too small, Step 2 of the algorithm quickly adjusted it to the proper size. CAPE 205 presently uses  $\epsilon_1 = \epsilon_2 = 10^{-4}$ ,  $\pi_0 = 2$ , and  $K = 10$ . Solutions typically examine 20 to 40 subproblems, many of which yield the same solution. The number of subproblems seems independent of the size of the problem. However, decreasing  $\epsilon_1$  or increasing  $K$  sharply increases the subproblems that are examined. In all cases, convergence to the optimum proceeded directly and quickly, and the subsequent location of a solution satisfying both constraints, if one existed, was similarly orderly and efficient.

## COMPUTATIONAL TESTING AND EVALUATION

### General

The Computer-Assisted Policy Evaluation System has been evaluated by solving a number of problems and performing a variety of preliminary analyses. These analyses provide in-depth insights into the effects of current policy and provide the cornerstone for integrated development of subsequent policy evaluation testing. Additionally, these analyses should have important implications for the design of subsequent policy and planning models and associated evaluation routines. The remainder of this section describes the scope and analysis of the computational testing and evaluation which was conducted.

Three primary policy areas, each representing a distinct type of policy, were subjected to extensive testing. These areas involve (1) the multiattribute facet of the assignment process, (2) the preemptive fill policy, and (3) a major billet eligibility policy.

To explore these areas, the input and model construction phases of the CAPE System were used to generate 15 sample problems from the Student Avail File and the LANT, PAC, and BUPERS Requisition Files for the May 1974 accounting period. The characteristics of these problems are summarized in Table 1. The problems are identified by group, rate, and pay grades, and the size of each problem is stated in terms of the total number of men, billets, and eligible man/billet combinations. In addition, the total number of nodes and arcs in the model of Figure 3, including circulation nodes and arcs, are indicated. Finally, Table 1 gives for each problem the three optimal functional values  $c^*$ ,  $d^*$ , and  $u^*$  obtained by solving for the minimum value of each of the functions independent of the others.

### Multiattribute Assignment

More precisely, analysis of the first policy area involves comparing the extended goal programming approach with the composite approach. Consequently, this entails an evaluation of the a priori weights for the utility, desirability, and cost functions (to obtain a single attribute coefficient for each nomination) versus using the extended goal programming model (to obtain the appropriate weights for a given proportionality criterion). Variances in weights associated with a given proportionality criterion disclose the extent to which the current state-of-the-art multiattribute a priori weighting approaches are inadequate for coping with multiattribute assignment problems. By pinpointing these variances and their relative magnitudes, it is possible to trace the consequences of present policy evaluation and disclose trade-offs afforded by better and more efficient solution procedures for the extended goal programming model.



Table 1

## Problem Set Characteristics

Problem	Group	Rate	Pay Grade	No. Men	No. Billets	Man/Billet Combinations	Total Nodes	Total Arcs	c*	d*	u*
1	01	2100	7	15	12	135	31	193	1976	4138	1994
2	01	2100	6	21	12	196	37	266	227	6906	635
3	01	2100	5	9	9	45	22	85	1049	907	1031
4	02	7400	6	27	9	117	40	193	1035	1254	2028
5	02	7400	5	30	18	280	52	380	1019	8390	4329
6	02	7400	4	9	9	51	22	91	2806	3246	3584
7	03	8000	8	21	14	205	39	279	167	457	4499
8	03	8000	7	93	41	1431	138	1703	11227	23006	14444
9	03	8000	6	114	23	838	140	1114	6937	13300	6890
10	03	8000	5	107	47	3145	158	3457	1075	17824	7479
11	15	8452	5	19	14	66	30	122	1215	3275	1556
12	22	8483	4	24	17	51	45	136	14221	14904	14767
13	22	8483	1-3	45	33	363	77	513	22917	28704	21997
14	32	8501	4	18	14	105	36	173	5904	8619	7254
15	32	8501	1-3	14	5	35	22	75	1948	1357	958



To identify these variances, each of the 15 sample problems was solved three times using each of the cost, desirability, and utility functions independently to obtain the  $c^*$ ,  $d^*$ , and  $u^*$  values given in Table 1. These problems were then solved using the a priori approach with an equal set of weights (i.e., using the composite objective function obtained by forming a weighted linear combination of the cost, desirability, and utility functions, with all weights equal to unity). The functional deviations from optimality were next computed using the a priori results to determine if the deviations were consistent. These results are given in Table 2. The quantities  $s_1$  and

$s_2$  represent the amounts by which the constraints on functional deviations from optimality are satisfied and are defined by

$$s_1 = \alpha - 1 - \alpha u^1 x + c^1 x$$

$$s_2 = \beta - 1 - \beta d^1 x + c^1 x.$$

Thus, a positive (negative)  $s_i$  indicates the corresponding constraint is satisfied (unsatisfied). As indicated by the  $s_1$  and  $s_2$  columns, both con-

straints were satisfied in only four of the 15 cases using this weighting scheme. The columns labeled  $cx$ ,  $dx$ , and  $ux$  give the values of the cost, desirability, and utility functionals, respectively, for the optimal solution.

Next, the 15 problems were solved using the extended goal programming model and the same set of proportionality weights. With this approach, solutions satisfying both constraints were found for 11 of the 15 problems. The results of solving the extended goal programming model are also given in Table 2. In addition, the columns  $y_1$ ,  $y_2$ , and  $y_3$  give the optimal composite objective function weights for the cost, utility, and desirability functions, respectively. In other words, if the problems were solved using the composite objective function  $(y_1 c + y_2 u + y_3 d)x$ , the solutions given in Table 2 would be obtained.

These problems were also solved with both approaches using unequal weights. In these examples, it was desired to give the utility function one-half the weight of the cost function and to give the desirability function twice the weight of the cost function. The a priori weighted composite objective function is thus  $(c + 1/2u + 2d)x$ . The results of these solutions are given in Table 3. This time, the a priori weighting scheme satisfied the constraints in eight of the problems, while the EGP model found 12 solutions which were consistent.

Thus, in a preponderance of the cases tested, the extended goal programming approach finds a solution satisfying both constraints, whereas the a priori model does not. Where consistent solutions were found using the a priori approach, the EGP model usually yielded solutions which lessened the functional deviations, most notably the cost component. In these cases, the  $cx$  value was never increased using the EGP technique and was usually decreased. Thus, the EGP technique was empirically verified on all measures to be substantially more effective than the a priori technique previously used.

Table 2

Comparison of EGP Model and A Priori Weighting With Equal Weights

Problem	A PRIORI WEIGHTING (1,1,1)					EXTENDED GOAL PROGRAMMING MODEL							
	s <sub>1</sub>	s <sub>2</sub>	cx	dx	ux	s <sub>1</sub>	s <sub>2</sub>	y <sub>1</sub>	y <sub>2</sub>	y <sub>3</sub>	cx	dx	ux
1	-.309	-.288	2096	5583	2731	.252	.255	1.0	1.578	1.038	2955	5131	2480
2	-.360	.184	300	7855	1068	3.980	3.987	1.0	9.772	1.248	1159	7724	715
3	-.794	.025	1075	907	1875	.619	.325	1.0	3.278	2.475	1923	1368	1252
4	-.033	.052	1089	1254	2200	.006	-.051	1.0	.597	.227	1096	1393	2135
5	-.137	-.014	1073	8949	5153	.796	.824	1.0	4.006	1.501	1953	9164	4851
6	-.018	.032	2896	3246	3763	-.018	.032	1.0	1.053	.291	2896	3246	3763
7	.213	-1.602	228	1356	5184	5.280	5.587	1.0	.00002	.992	1100	457	5879
8	-.041	.005	11359	23159	15213	.086	.059	1.0	11.285	5.141	12313	23867	14601
9	.013	.010	7028	13340	6890	.006	-.002	1.0	.160	.175	7002	13450	6911
10	-.265	.177	1275	17985	10855	.731	.706	1.0	6.302	1.982	2086	22002	9048
11	.366	.367	1694	3363	1600	.366	.367	1.0	.711	.009	1694	3363	1600
12	.026	.057	15034	14904	15228	.018	.035	1.0	.574	.372	14918	15106	15228
13	.021	.025	23496	28704	22083	.021	.025	1.0	.963	.229	23496	28704	22083
14	-.023	-.012	5904	8726	7419	.053	.145	-1.0	1.371	.735	6901	8825	8093
15	-1.486	.045	2035	1357	2424	-.304	.461	1.0	1.112	.942	2846	1357	1691

Table 3

Comparison of EGP Model and A Priori Weighting With Unequal Weights

Problem	A PRIORI WEIGHTING (1, 1/2, 2)					EXTENDED GOAL PROGRAMMING MODEL							
	s <sub>1</sub>	s <sub>2</sub>	cx	dx	ux	s <sub>1</sub>	s <sub>2</sub>	y <sub>1</sub>	y <sub>2</sub>	y <sub>3</sub>	cx	dx	ux
1	.612	.942	3838	4138	3312	.173	.164	1.0	0.0	1.256	2934	4802	3237
2	3.769	3.992	1160	7315	1068	-.019	.047	1.0	.740	1.060	300	7855	1068
3	-.385	.025	1075	907	1875	1.514	.636	-1.0	4.170	6.262	2782	1368	1316
4	.010	.052	1089	1254	2200	-.024	.032	1.0	.0003	.958	1068	1254	2253
5	-.077	-.109	1044	8949	5204	.806	.850	1.0	.320	2.768	1937	8605	5154
6	.007	.032	2896	3246	3763	.007	.032	1.0	.108	8.747	2896	3246	3763
7	5.543	5.659	1112	457	5544	5.433	5.587	1.0	.000006	.991	1100	457	5879
8	-.026	.016	11357	23006	15521	.066	.095	1.0	12.037	28.571	12315	23032	15342
9	.013	.007	7028	13340	6890	.011	.006	1.0	.107	.660	7024	13340	6911
10	-.092	.164	1251	17824	11297	.826	.796	1.0	5.844	2.242	2137	19537	9894
11	.380	.340	1694	3363	1600	.309	.311	1.0	.032	.954	1658	3363	1730
12	.042	.057	15034	14904	15228	.030	.051	1.0	.072	.952	14947	14904	15403
13	-.018	0.0	22917	28704	22791	-.018	0.0	1.0	.053	.252	22917	28704	22791
14	-.035	.001	5911	8619	7783	.087	.147	-1.0	.813	4.294	6908	8718	8457
15	-.720	.045	2035	1357	2424	.211	.594	-1.0	.832	2.347	3105	1357	1691



However, one limitation of the EGP procedure is illustrated by problem 9, Table 2, and problems 2 and 4, Table 3. In those cases, the solutions obtained by the a priori weighting satisfied the constraints, but overly so, thereby yielding feasible solutions of poor quality. The new model found solutions which came closer to satisfying both of the constraints with equality, and thus which were of superior quality, but at the expense of not quite satisfying one of the constraints. This was a result of the termination of the search procedure at a local optimum as defined by the Lagrangean functional. This shortcoming of such approximation techniques prohibits guaranteeing satisfaction of the constraints and points out the need for a solution algorithm which can handle these restrictions explicitly as originally anticipated. The situation is also aggravated by the small problem size where fewer integer solutions lie close to the Lagrangean optimal. It should be reemphasized, however, that in these test cases the near-satisfactory solutions produced savings in terms of the cost function.

In the 18 cases where the a priori weighting solution violated the constraint set, the EGP method was notably superior, finding 14 solutions satisfying both constraints and two solutions with improved deviations. In only two of the 18 cases were the solutions obtained by the two approaches the same. An interesting phenomenon was the discovery that satisfaction of the constraints on functional deviations from optimality may be somewhat expensive, as illustrated by problems 2 and 7, Table 2, and problem 3, Table 3. These high costs may also be caused by the small number of feasible solutions in the vicinity of the Lagrangean optimal and by the inability of the out-of-kilter method to generate alternative optima for evaluation in terms of the cost functional. This type of limitation is related to the one noted earlier and can be overcome by a procedure that has an ability to handle side-constraints explicitly.

One point is made very strongly by the  $y_1$ ,  $y_2$ , and  $y_3$  values given in Tables 2 and 3: these optimal weights are quite different from the a priori weights. The values range from -1.0 to 11.285 in the cases using the (1,1,1) weight set and from -1.0 to 28.571 when the (1, 1/2, 2) weights are used. Rare is the case in which the optimal weights resemble the a priori weights. Note also that, in four problems, the  $y_1$  value is in fact negative, the somewhat counter-intuitive outcome which was described previously.

Thus, these findings demonstrate that the plausibility of the arguments underlying the a priori approach cannot be supported empirically, i.e., the more rigorous mathematical foundations of the EGP approach lead to important practical as well as theoretical differences. The composite objective function technique used by CADA clearly does not guarantee that the desired relative significance of the competing goals of cost, utility, and desirability will be enforced in the optimum solutions to the manpower assignment problems. The EGP approach, on the other hand, can ensure that the desired policies will be observed.

To compare the extended goal programming approach with the surface optimization approach (Hatch et al., 1972), we solved the set of 15 test problems subject to two different absolute hierarchical rankings of the goals. In the first case, cost was given the highest ranking, followed in order by desirability and utility. The second ranking examined was desirability, cost, and utility. Table 4 gives the results for both rankings. The columns  $cx$ ,  $dx$ , and  $ux$  give the functional values in the optimal solution, and the  $s_1$  and  $s_2$  columns show the functional deviations from optimality and are defined as above. Comparing these deviations then provides a measure of the adequacy of the surface optimization approach to cope with multiple objectives.

Table 4

## Surface Optimization

Problem	Ranking: c, d, u				Ranking: d, c, u			
	cx	dx	ux	$s_1$	$s_2$	cx	dx	ux
1	1976	7474	2656	-.332	-.806	3821	4138	3635
2	227	8976	1595	-1.512	-.298	2006	6906	1406
3	1049	1805	1292	-.253	-.990	1070	907	2061
4	1035	4327	2358	-.163	-2.451	1068	1254	2253
5	1019	10004	5122	-.183	-.192	2825	8390	5218
6	2806	3246	4608	-.286	.000	2806	3246	4608
7	167	4952	5383	-.196	-9.836	1100	457	5879
8	11227	25037	16863	-.167	-.088	11305	23006	16654
9	6937	14569	7959	-.155	-.095	7891	13300	7801
10	1075	23730	12896	-.724	-.331	1177	17824	7479
11	1215	3807	2352	-.512	-.162	2097	3275	1558
12	14221	16005	15912	-.078	-.074	14947	14904	15403
13	22917	28704	22791	-.036	.000	22917	28704	22791
14	5904	8726	7419	-.023	-.012	5911	8619	7783
15	1948	2255	3420	-2.601	-.662	2035	1357	2424

For the (c,d,u) ranking, since cost is the top ranked goal and the problem is first optimized with respect to c, then  $cx = c^*$  (see Table 1). Therefore, the deviations of the desirability and utility functionals from their respective optima must exceed the deviation of cx from its optimum  $c^*$ . This result is verified by Table 4 where we see that, except for two cases, the  $s_1$  and  $s_2$  values are all negative. In fact, in only relatively few of the problems do the  $s_1$  approach zero. This is the very serious defect inherent in the surface optimization approach. Once the top ranked goal has been accommodated, there is little chance that the second and third objectives can be accommodated to any significant degree.

The results for the (d,c,u) ranking confirm these shortcomings. Since d is the top ranked function, the  $s_2$  are all nonnegative. However, the entries in the  $s_1$  column show wide fluctuations, from -1.486 to 6.623. Clearly, once the problem has been optimized with respect to the desirability function alone, giving  $dx = d^*$ , very little else can be accomplished with respect to the cost and utility functions. In some cases, such as problems 3, 4, 6, 8, 13, and 14, the value of cx happens to be close to  $c^*$ , but in other cases, most notably problems 2, 5, and 7, cx is very much greater than  $c^*$ . Also, in those cases where cx is close to  $c^*$ , there is even less chance that ux will be close to  $u^*$ , and we get  $s_1$  much less than zero (see problems 3 and 15, especially).

In general, for both rankings in Table 4, comparing the functional values for the second and third ranked goals with the appropriate optima in Table 1 vividly illustrates the failure of the surface optimization approach to achieve these secondary goals. The primary goal can be accommodated completely, but the secondary goals are left to suffer the consequences. Therefore, surface optimization is clearly inadequate in situations where multiple goals have some meaningful relative significance and deserve more than token consideration--as in the manpower assignment context.

#### Preemptive Fill Policy

The second policy area investigated was the preemptive policy of filling the maximum number of billets regardless of the utility, desirability, and cost functions. The 15 test problems were first solved utilizing the preemptive policy to determine the maximum number  $m^*$  of billets that can be filled (see Table 5). The test problems were then solved six additional times using the following combinations of percentage fill policies  $\gamma$  and costs KOST for not filling a billet:

	$\gamma$	KOST
1.	90%	999
2.	75%	999
3.	50%	999
4.	50%	1999
5.	0%	999
6.	0%	1999



Table 5  
Problem Solutions Using the Maximum Fill Policy

Problem	Men	Billets	Maximum Number of Billets Filled
1	15	12	12
2	21	12	12
3	9	9	9
4	27	9	9
5	30	18	18
6	9	9	7
7	21	14	14
8	93	41	30
9	114	22	17
10	107	47	47
11	19	7	6
12	23	17	3
13	40	33	11
14	18	14	9
15	13	5	5

The results of this testing were enlightening. For all problems except problem 14, all six of the above policies filled the maximum number of billets. In problem 14, policies 1, 2, 3, and 5 filled only 8 billets, but policies 4 and 6 both filled the maximum number, 9. For the different numbers of billets filled, Table 6 summarizes the changes in the cost, utility, and desirability functions and in the composite objective function  $z = (y_1c + y_2u + y_3d)x$  derived from the solution of the EGP model. Although there is a decrease in average cost per billet filled, it is clear that imposing the maximum fill policy increases the average penalty per billet filled for the utility and desirability functions and for the composite objective function from the EGP model. Finally, it was subsequently determined that the marginal cost for filling the ninth billet in problem 14 was in fact 1000.

Table 6

## Problem 14 Solution Using Billet Fill Policies

Billets Filled	cx		ux		dx		z	
	Total	Ave.	Total	Ave.	Total	Ave.	Total	Ave.
8	907	113	2099	262	2831	354	4052	506
9	909	101	2424	267	3731	415	5057	574

Next an examination of the remaining problems was undertaken to investigate the lack of effect of varying the values of  $\gamma$  and KOST. Obviously, in some of the problems (e.g., problems 1-5), the reason is simply that all billets are filled. However, in most of the other problems, there are large numbers of unfilled billets. A visual examination of several of these examples revealed the following interesting problem structure. Both the set of men and the set of billets can be partitioned into disjoint subsets. Further, the men in a given subset are eligible for assignment to billets in only one of the subsets of billets. Unfilled billets are the result of having subsets of men which are too small to fill all the billets in the subset of billets for which they are eligible. An additional feature of most of the problems is the existence of men who are not eligible for assignment to any of the billets. Table 7 illustrates this eligibility structure for problem 12, one of the worst cases.

Table 7

## Assignment Eligibilities for Problem 12

Men (N=23)	Eligible Billets (N=17)
1,3,4,5,7,8,9,10,11,12, 13,14,15,18,19,20,21,23	1
6,17	2,3,4,5,6,7,8,9,10, 11,12,13,14,15,16,17
2,16,22	(not eligible for any billets)

Note: Maximum Possible Assignments: 3

These discoveries prompted a closer examination of the effects of the man/billet eligibility policies and the rules for computing the cost, desirability, and utility factors as implemented by CADA 203 (and likewise used by CAPE). The most apparent problem concerned the computation of the components of the desirability function. Each man in the Student Avail File can specify three preferences for type of duty and for location. There appear to be two serious shortcomings of this data. In many cases the preferences are not specified, or only incompletely specified. Secondly, in most instances where the billet preference information is stated, there is no correspondence with the actual billets for which the man is being considered. In other words, the location and type of duty of the billets for which the individual is actually eligible do not relate to the preferences indicated by the individual. Both of these problems produce a situation in which all billets are equally undesirable to an individual. Therefore, the meaningfulness of the desirability function is questionable. Next, we attempted unsuccessfully to isolate the eligibility rules which cause the problem structures indicated in Table 7. It is interesting to note, however, that in the larger problems, an individual is eligible, on the average, for only one-third to one-half of the billets in his rate and pay grade. Clearly a detailed investigation of the effects of the various eligibility rules and an attempt to identify those that may be unduly restrictive should be undertaken.

In summary, it appears that the policy of maximum personnel assignments has little effect on problem solutions. As long as the value of KOST is 999 (i.e., somewhat larger than the maximum value, 900, of the cost, utility, and desirability functions), the maximum possible number of billets will be filled. The large numbers of unfilled billets are inherent in the man/billet eligibility rules employed by CAPE 203 (which are taken directly from CADA 203).

#### Billet Eligibility Policy

The last policy area, on which very preliminary testing was conducted, deals with projected rotation date (PRD). Currently, enlisted personnel are considered eligible for transfer only if their PRD does not exceed the current accounting period date by more than 5 months. Varying the length of this eligibility "window" greatly affects the number of men eligible for assignment. To test the effects of variations in the length of this available period, three problems, 8, 10, and 13, were selected from the original set of sample problems and were regenerated using eligibility windows of 4, 3, and 2 months. Periods longer than 5 months were not considered since the Student Avail File for May 1974 accounting period had very few men with PRD's 6 or more months after this date.

The results of solving these three problems utilizing a preemptive maximum fill policy are summarized in Table 8 and presented graphically in Figure 5. Readily apparent are the significant reductions in the number of available men as the eligibility "window" is reduced from 5 to 2 months. These reductions are clearly not uniform, with problems 8 and 10 undergoing 70% and 71% reductions, respectively, and problem 13 experiencing only a 20% decrease in the number of available individuals. These decreases are compared to a corresponding reduction in the total number of available men in the May 1974 file from 1829 to 503, or 73%.



Table 8

## Problem Solving Using PRD Policy

Problem	Max. Months PRD Follows CAPE	Men	Billets	Man/Billet Combinations	Max. No. of Billets Filled	cx	ux	dx
8	5	93	41	1431	30	11357	15521	23006
8a	4	68	41	1034	28	13387	16588	24199
8b	3	36	41	516	22	19255	21984	27293
8c	2	28	41	374	20	21161	23952	29017
10	5	107	47	3145	47	2086	9048	22002
10a	4	89	47	2638	46	2179	9652	23088
10b	4	57	47	1592	44	5219	9039	29175
10c	2	31	47	834	27	21178	23192	33668
13	5	40	33	363	11	23496	22083	28704
13a	4	40	33	363	11	23496	22083	28704
13b	3	35	33	198	6	27443	27065	30575
13c	2	32	33	99	3	29973	29981	29973

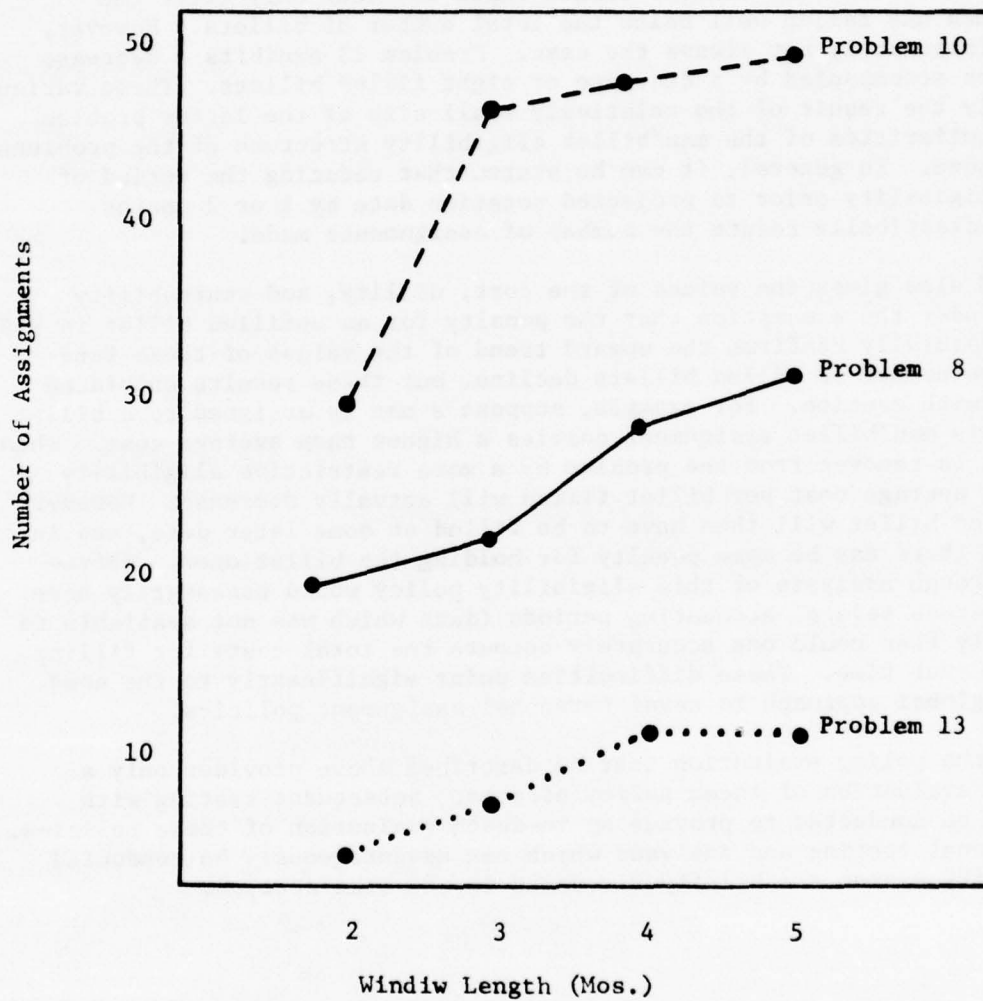


Figure 5. Problem solutions using PRD policy.

Since many of the sample problems involve more men than billets, one would expect that a reduction in the number of men would yield a smaller decrease in the number of billets filled. This is confirmed in problems 8 and 10. In the first case, a decline of 25 men from problem 8 to problem 8a causes a loss of two filled billets. Comparing problems 10 and 10b shows that a reduction of 50 men produces the small loss of only three filled billets. Significant declines in the number of billets filled begin to occur only after the number of men has fallen well below the total number of billets. However, this is unfortunately not always the case. Problem 13 exhibits a decrease of eight men accompanied by a decrease of eight filled billets. These variances are probably the result of the relatively small size of the latter problem and the peculiarities of the man/billet eligibility structure of the problems as noted above. In general, it can be stated that reducing the period of transfer eligibility prior to projected rotation date by 1 or 2 months should not drastically reduce the number of assignments made.

Table 8 also gives the values of the cost, utility, and desirability functions under the assumption that the penalty for an unfilled billet is 999. This data naturally confirms the upward trend of the values of these functions as the number of filled billets decline, but these results should be considered with caution. For example, suppose a man is assigned to a billet and that this man/billet assignment carries a higher than average cost. Then, if this man is removed from the problem by a more restrictive eligibility policy, the average cost per billet filled will actually decrease. However, this unfilled billet will then have to be filled at some later date, and in the interim there may be some penalty for holding the billet open. Therefore, a thorough analysis of this eligibility policy would necessarily have to extend across several accounting periods (data which was not available to us); for only then could one accurately compute the total costs for filling all billets over time. These difficulties point significantly to the need for a more global approach to naval personnel assignment policies.

All of the policy evaluation testing described above provides only a preliminary evaluation of these policy effects. Subsequent testing with CAPE should be conducted to provide an in-depth evaluation of these policies. Such additional testing and analyses which can advantageously be conducted using the CAPE system are briefly discussed in the next section.



## CONCLUSIONS AND RECOMMENDATIONS

This section provides a partial list of policy questions (scenarios) for which the CAPE system and/or the extended goal programming model should be applied.

1. CAPE can be used to test the effect of varying the formulas for calculating the parameters of the utility and desirability functions. In particular, the EGP model can be expanded to include each parameter as a separate function.

2. CAPE can be used to test the effect of allowing the proportionality weights  $\alpha$  and  $\beta$  to be parameters in the model. This would provide policymakers with an idea of the influence of attaching different degrees of importance to each goal.

3. CAPE can be used to test the current critically important questions posed in the Forward Plan for NEOCS. That is, must the growth of NEC ratings be stopped? Results reported in this plan (which are based on qualitative interviews) indicate that the increase in the number of NEC rates must be stopped or the present manual personnel assignment system will be seriously crippled. This could be evaluated by grouping similar NEC ratings and using CAPE to solve the resulting problems.

4. An evaluation of the billet fill priority system can be conducted. Specifically, each requisition displays the priority order in which the Manning Control Authorities (MCA) desires to have the vacancy filled. This priority system includes preemptive priority classifications, namely a MUST FILL priority and priorities 1 and 2 assigned by the Chief of Naval Operations. The effect of these preemptive priority classifications should be determined since the NEOCS plan reported that these priorities substantially complicate efficient detailing of personnel.

5. Most billet and assignment rotation eligibility policies could be evaluated using CAPE.

All of these evaluations promise to yield significant information which is presently lacking. Items 3 and 4 are especially crucial and should be undertaken if effective policy formulation is to be carried out.

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APPENDIX  
CAPE 204 AND 205  
PROGRAM DOCUMENTATION



CAPE 204  
PROGRAM DOCUMENTATION

I. IDENTIFICATION

Title: CAPE 204  
Classification: Problem generator  
Source Language and Type: COBOL program  
Program and Documentation: David Karney  
Organization: Center for Cybernetic Studies, The University of  
Texas at Austin

II. INTRODUCTION

CAPE 204 accepts the list of eligible man/billet combinations, file CAPETD, generated by CAPE 203 and produces the corresponding complete extended goal programming problems on CAPEPR to be solved by CAPE 205. CAPETD contains problem identification and the individual components of the three objective functions--cost, utility, and desirability--which have been standardized to have a mean and standard deviation equal to one. CAPE 204 produces the appropriate title and keyword cards, forms weighted sums of the individual components yielding single values for cost, utility, and desirability, and normalizes these values to lie in the range from 1 to 900. In addition, the required circularization and other special nodes and arcs are generated. The result is the complete multiobjective function pure network flow problem which represents the extended goal programming model for the personnel assignment problem. CAPE 204 also logs the problem data on the file CAPETG used as input to the solution analysis routine CADA 206.

III. OPERATING PROCEDURES

CAPE 204 is a stand-alone program which produces the output files CAPEPR and CAPETG by the appropriate processing of the input file CAPETD and the set of component weighting factors obtained from the input file CAPEWT. No other input or user-specified action is required. Figure 1 gives a sample input deck structure for CAPE 204.

IV. INPUT

CAPE 204 uses two input files CAPETD and CAPEWT. CAPETD is generated by CAPE 203 and contains the list of eligible man/billet combinations. It consists of sets of records, each set defining the man/billet assignment eligibilities for a single rate-pay grade grouping. Each set of records contains a header record, the man/billet combinations, and a set of billet definition records. The format of each type of record is as follows:

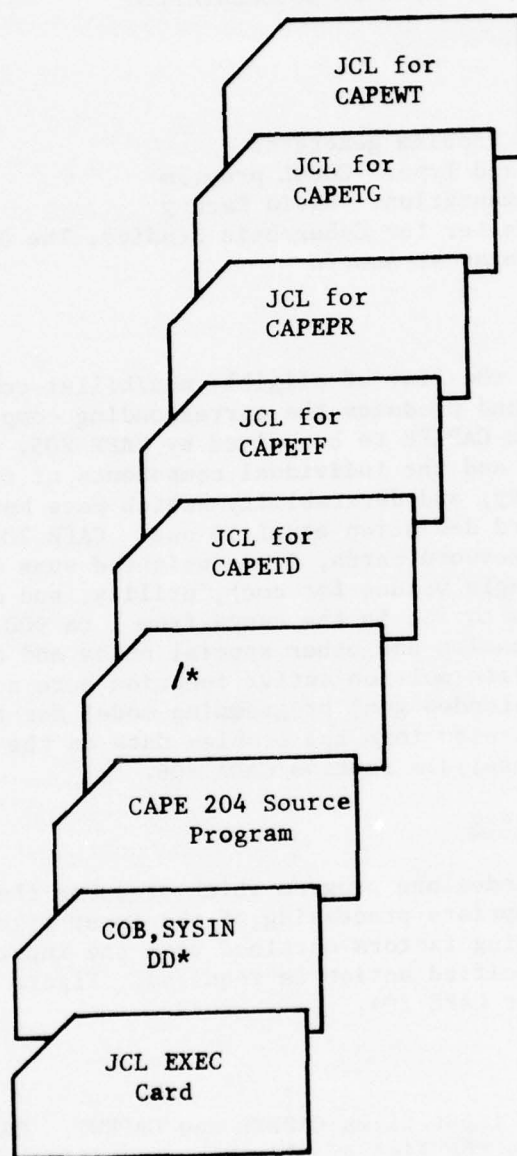


Figure 1. Sample Deck Setup for CAPE 204.

<u>Record Type</u>	<u>Character Positions</u>	<u>Data</u>
Header	1-8	Zeros
	9-12	Rate
	13	Lowest pay grade
	14	Highest pay grade
	15-16	Group code
<hr/>		
Man/Billet	1-4	Man identification
	5-8	Billet identification
	9	Force flag
	10	School code
	11-13	
	14-16	
	17-19	
	20-22	Raw scores for utility
	23-25	components (COMP-3)
	26-28	
	29-31	
	32-34	Raw scores for desir-
		ability components
		(COMP-3)
	35-37	Filler
	38-40	
		Raw score for cost
		component (COMP-3)
	41	Cost flag
	42-48	
	49-55	Utility function
	56-62	components, right-
	63-69	justified, four
	70-76	assumed decimal places.
	77-83	
	84-90	
	91-97	Desirability function
		components, right-
		justified, four assumed
		decimal places
	98-104	Filler
	105-111	
		Cost function component,
		right-justified, four
		assumed decimal places

---



<u>Record Type</u>	<u>Character Positions</u>	<u>Data</u>
Billet Definition	1-4	Zeros
	5-8	Billet identification

The logical records each contain 111 characters and are blocked 85 per physical record.

The input file CAPEWT contains the individual component weights used by CAPE 204 in generating the utility and desirability values. The utility and desirability functions are each composed of several individual components. CAPE 204 computes a weighted linear combination of these components yielding single functional values u and d for utility and desirability as follows:

$$u = \sum_{i=1}^6 \alpha_i u_i$$

$$d = \sum_{i=1}^2 \beta_i d_i$$

where  $u_i$  and  $d_i$  are the individual function components and the  $\alpha_i$  and  $\beta_i$  are the weights inputs from CAPEWT.

CAPEWT consists of card images each containing the weights for a single rate-pay grade grouping. The format of each record is as follows:

<u>Character Positions</u>	<u>Data</u>
1-4	NEC rate
5	Lowest pay grade in group
6	Highest pay grade in group
7-8	
9-10	
11-12	Weights for the six utility
13-14	function components, right-
15-16	justified, one assumed decimal
17-18	place
19-20	Weights for the two desirability
21-22	function components, right-justified,
	one assumed decimal place

The records in CAPEWT are unblocked 80 character card images and must be sorted by increasing and decreasing pay grade. Weights not specified are assumed to be unity. For example, if there is no record in CAPEWT corresponding to a given rate-pay grade group in CAPETD, all weights are assumed to be 1.0. Also, weights corresponding to blank two column fields in the records in CAPEWT are assumed to be 1.0.

#### V. OUTPUT

CAPE 204 generates the network problems on file CAPEPR. The man and billet nodes are named by prefixing the man and billet identification numbers with M and B, respectively. The cost, utility, and desirability components are combined and standardized to lie in the range 1-900. A special source node, MSSOR, and a special sink node, BSSNK, are added to accommodate the billet-fill policies. The arcs from each man to BSSNK, the arcs from MSSOR to each billet, and the arc from MSSOR to BSSNK are added. Finally, circulation nodes MMSOR and BMSNK and the necessary circulation arcs are also added. The problems are written as sets of card images according to the following format:

<u>Card No.</u>	<u>Column Nos.</u>	<u>Data</u>
1	1-5	BEGIN
2	1-80	Descriptive title card giving rate, paygrades, and group code
	57-61	Number of men
	73-77	Number of billets
3	1-4	ARCS
4-n		Arc data cards
	8-12	From node name
	14-18	To node name
	21-30	Cost function value, right-justified
	31-40	Upper-bound, right-justified
	41-50	Lower bound, right-justified
	51-60	Desirability function value, right-justified
	61-70	Utility function value, right-justified
n+1	1-3	END

The last problem in the file is followed by a trailer card containing "QUIT" in columns 1-4.

In addition, CAPE 204 generates the file CAPETG containing the raw data and the computed utility, desirability, and cost values for the eligible man/billet combinations. The file is organized in sets of records, each set containing the man/billet combinations for a single rate-pay grade grouping. Each set of records consists of a header record identifying the group and giving the component weights, followed by one record for each eligible man/billet combination. The format of each type of record is as follows:

<u>Record Type</u>	<u>Character Positions</u>	<u>Data</u>
Header	1-8	Zeros
	9-12	NEC Rate
	13	Lowest pay grade
	14	Highest pay grade
	15-16	Group Code
	17	Weights flag: 1 if weights were input for this group, 0 otherwise
	18-19	Weights for utility function components, right-justified, one assumed decimal place
	20-21	
	22-23	
	24-25	
	26-27	
	28-29	Weights for desirability function component, right-justified, one assumed decimal place
	30-31	
	32-33	
-----		
Man/Billet	1-4	Man identification
	5-8	Billet identification
	9	Force flag
	10	School code



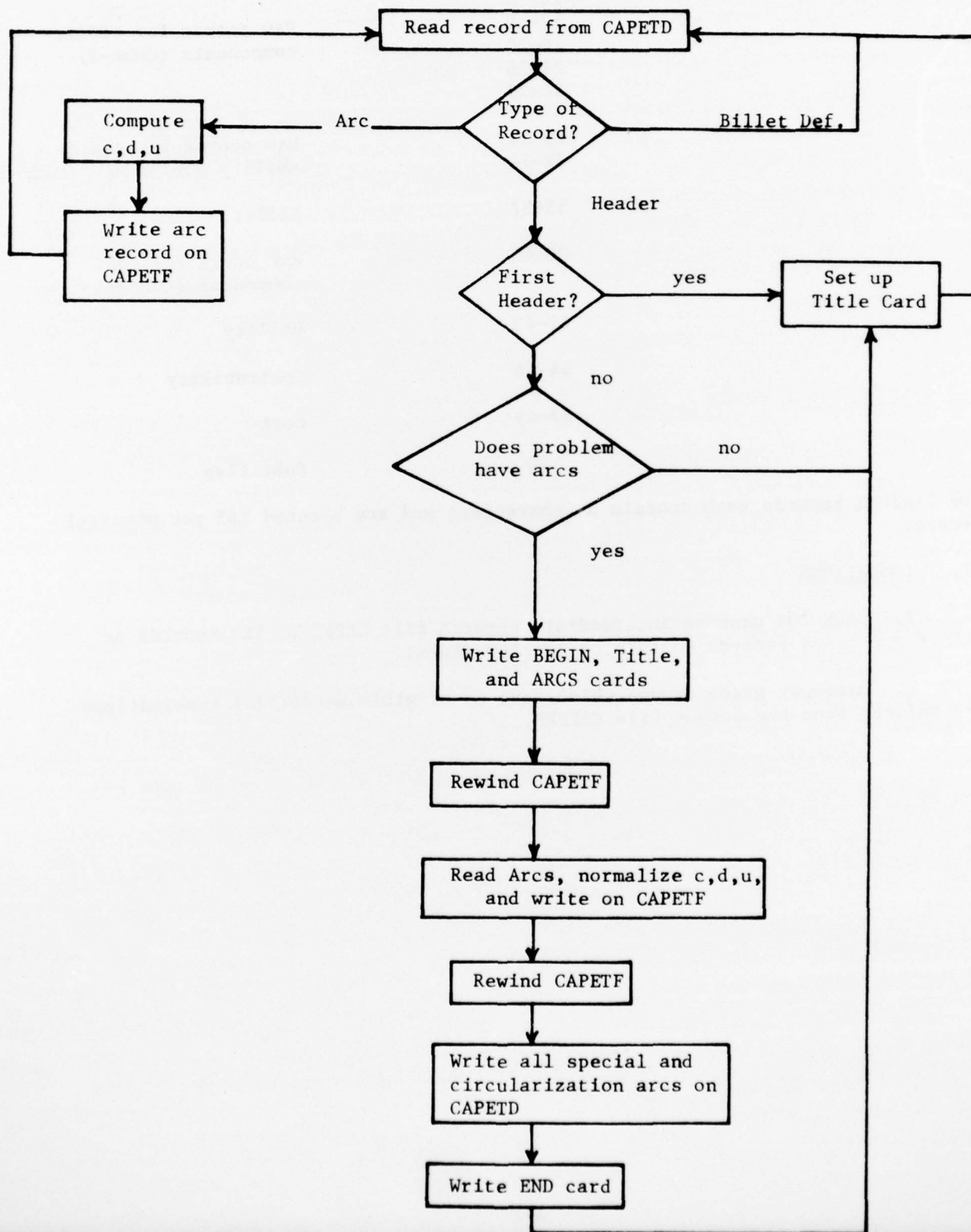
<u>Record Type</u>	<u>Character Positions</u>	<u>Data</u>
	11-13	
	14-16	
	17-19	Raw scores for utility
	20-22	components (COMP-3)
	23-25	
	26-28	
	29-31	Raw scores for desir-
	32-34	ability components (COMP-3)
	35-37	Filler
	38-40	Raw score for cost
		component (COMP-3)
	41-43	Utility
	44-46	Desirability
	47-49	Cost
	50	Cost flag

The logical records each contain 50 characters and are blocked 165 per physical record.

#### VI. CONDITIONS

1. CAPE 204 uses an intermediate scratch file CAPETF. The records on CAPETF are coded records containing 32 characters.
2. Rate-pay grade groups which have no eligible man/billet combinations are omitted from the output file CAPEPR.

VII. FLOWCHART



CAPE 205  
PROGRAM DOCUMENTATION

I. IDENTIFICATION

Title: CAPE 205  
Classification: Mathematical programming, multiobjective function  
pure network problems  
Source Language and Type: FORTRAN IV program  
Program and Documentation: David Karney  
Organization: Center for Cybernetic Studies, The University of  
Texas at Austin

II. INTRODUCTION

CAPE 205 solves the extended goal programming formulation of the multi-objective function personnel assignment problems generated by the CAPE system. These problems have three objective functions--cost, utility, and desirability--and are modelled as pure network flow problems having two additional linear constraints enforcing specified proportionalities in functional deviations from optimum. CAPE 205 employs Lagrangean search techniques and uses the highly efficient out-of-kilter network algorithm SUPERK to solve the associated pure network subproblems.

III. OPERATING PROCEDURES

CAPE 205 is a stand-alone program and solves the problems contained in the file CAPEPR generated by CAPE 204. The execution of CAPE 205 is controlled by user specified directions entered through a designated command file. An output file containing the problem solutions is produced. Figure 1 gives a sample input deck structure for using CAPE 205.

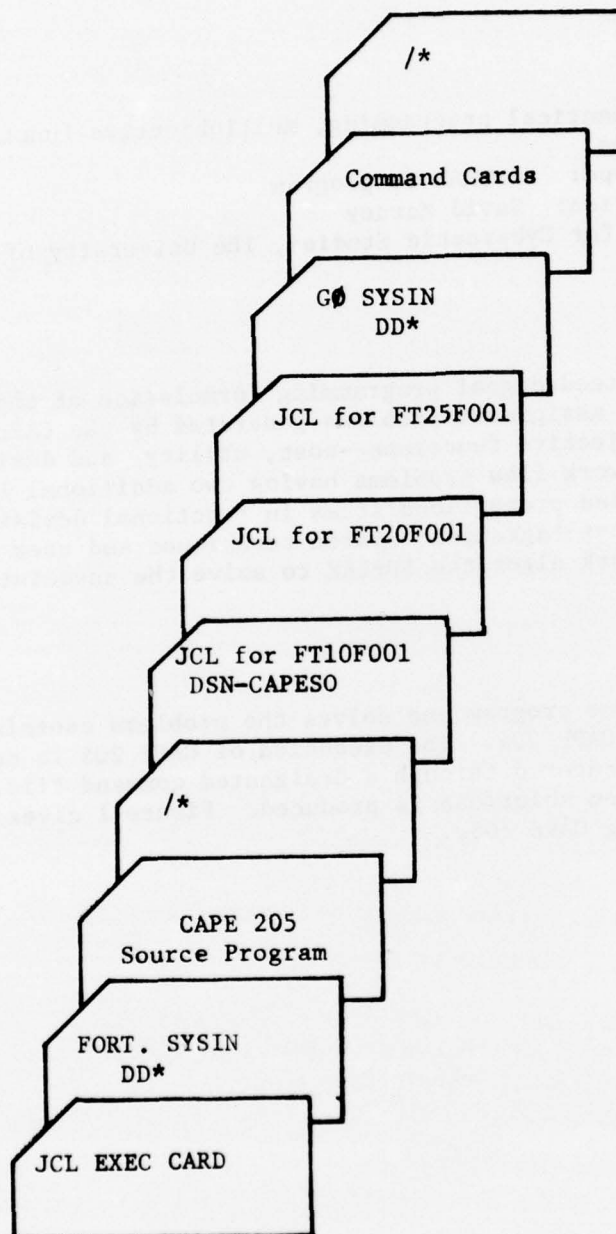


Figure 1. Sample deck setup for CAPE 205.



#### IV. INPUT

CAPE 205 uses two input files: the problem file CAPEPR generated by CAPE 204 and the command file through which the user controls the execution of the program. CAPEPR contains sets of card images, each set defining one independent problem according to the following format:

<u>Card No.</u>	<u>Column Nos.</u>	<u>Data</u>
1	1-5	BEGIN
2	1-80	Descriptive problem title with the number of men in columns 57-61 and the number of billets in columns 73-77
3	1-4	ARCS
4-n		Arc data cards
	8-12	From node name
	14-18	To node name
	21-30	Cost function value, right-justified
	31-40	Upper bound, right-justified
	41-50	Lower bound, right-justified
	51-60	Desirability function value, right-justified
	61-70	Utility function value, right-justified
n+1	1-3	END

After the last problem there is a trailer card containing "QUIT" in columns 1-4.

The command file entries are card images containing a command word followed by zero, one, or two arguments. Commands are assumed to be left-justified in column 1-6 of a card. Arguments are specified as integers or decimal numbers separated by one or more blanks and may be anywhere in columns 7-80. In the following description, command words are capitalized, lun denotes a logical unit number, n denotes an unsigned integer, and x denotes a decimal floating point number. Appropriate default values are also indicated.

##### Command

1. LUCOMM lun      Set command file logical unit number to lun. Default is lun = 1.
2. LUDATA lun      Set problem input data file logical unit number to lun. Default is lun = system input file (5).

- |     |            |   |
|-----|------------|---|
| 3.  | LUOUT lun  | Set solution output data file logical unit number to lun. Default is lun = 10.  |
| 4.  | REWIND lun | Rewind file lun.  |
| 5.  | SKIP n     | Skip n problems on the problem input data file. A problem is treated as a set of card images ending with an END card.   |
| 6.  | SOLVE n    | Read and solve n problems from the problem input data file.   |
| 7.  | REPORT n   | Set report flag IREPT = n. If IREPT = 1, print complete solution including all arcs regardless of flow and all circulation arcs. If IREPT = 2, print solution giving only man/billet arcs having non-zero flow. |
| 8.  | STOP       | Terminate run.  |
| 9.  | ALPHA x    | Set proportionality weight $\alpha = x$ . Default is $\alpha = 1.0$ .   |
| 10. | BETA x     | Set proportionality weight $\beta = x$ . Default is $\beta = 1.0$ .   |
| 11. | GAMMA x    | Set percentage maximum billet-fill $\gamma = x$ . Default is $\gamma = 1.0$ .   |
| 12. | KOST x     | Set cost of unfilled billet KOST = x. Default is KOST = 999.  |

The following is a sample command stream which will solve problems 1, 4, and 5 of a data set with different values for  $\alpha$  and  $\beta$ .

<u>Command</u>	<u>Action</u>
SOLVE 1	Solve first problem in data set using default values for all parameters.
SKIP 2	Skip next two problems.
SOLVE 2	Solve next two problems.
REWIND 5	Rewind data file.
ALPHA 2.0	Set $\alpha = 2.0$ .
BETA 0.5	Set $\beta = 0.5$ .
REPORT 2	Set report flag to print all assignments made.
LUOUT 20	Change solution output file to lun 20.
SOLVE 1	Re-solve first problem with new parameter values.
SKIP 2	Skip two problems.
SOLVE 2	Solve two problems.
STOP	Stop run.

## V. OUTPUT

CAPE 205 produces a printed report for each problem solved giving the title card; the number of nodes and arcs in the problems; the current policies ( $\alpha$ ,  $\beta$ ,  $\gamma$ , and KOST); the maximum number of billets that can be filled  $m^*$ ; the optimal functional values  $c^*$ ,  $u^*$ , and  $d^*$ ; the functional values, functional deviations from optimality, and the number of billets filled when using the a priori weighted sum of the objective function; and the functional values, functional deviations from optimality, the number of billets filled, and the optimal proportionality weights derived from the solution to the extended goal programming model. All user specified commands are logged on the printed output.

CAPE 205 also generates a file CAPESO containing the man/billet nominations obtained by solving the extended goal programming model. The solutions are written as sets of line images, each set giving the optimal assignments for one problem according to the following format:

<u>Line No.</u>	<u>Character Positions</u>	<u>Data</u>
1	1-12	Zeros
	13-16	NEC Rate
	17	Lowest pay grade
	18	Highest pay grade
	19-20	Group code
	21-25	$\alpha$ , weight for utility function (F5.2)
	26-30	$\beta$ , weight for desirability function (F5.2)
2-m		Optimal man/billet nominations
	2-6	Man (Mxxxx)
	8-12	Billet (Bxxxx)

## V. CONDITIONS

1. CAPE 205 uses several files for which default logical unit numbers are specified in the program's main routine using simple arithmetic replacement statements. These unit numbers can be changed as necessary; file usage is given below.

<u>Statement</u>	<u>File</u>
LUCARD = 5	System input file
LUPRNT = 6	System output file
LUCOMM = 1	Command file
LUOUT = 10	Problem solution output data file
LUSCR = 25	Intermediate scratch files used by CAPE 205
LUSCR2 = 20	

2. Maximum problem size that can be solved may be changed by setting two parameters in the main routine and by specifying the proper dimensions for the arrays in the labelled COMMON block/DATA/. The two problem size parameters are set by the following arithmetic replacement statements:

<u>Statement</u>	<u>Purpose</u>
MAXA = ....	Maximum allowable number of arcs
MAXB = ....	Maximum allowable number of nodes

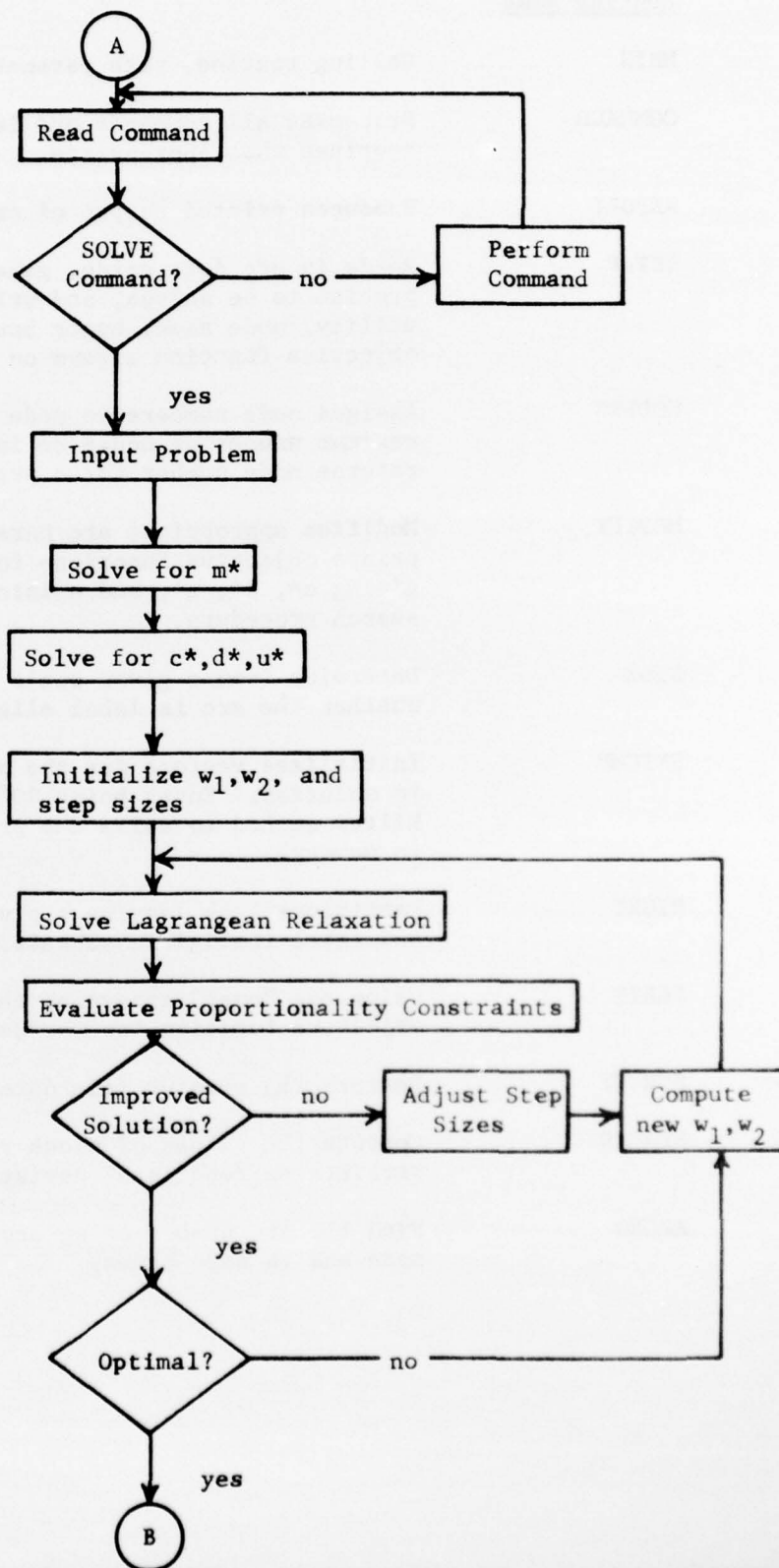
The following arrays in the labelled COMMON blocks/DATA/must be dimensioned for at least the following sizes:

<u>Array</u>	<u>Minimum size</u>
NODE,MIDL,LABL,IWV, IWV2	MAXN + 1
MIR,NA,NC	2*MAXA+1
KOS,JWV	MAXA + 1

3. The two scratch files used by CAPE 205 are read and written sequentially using unformatted FORTRAN binary I/O operations.



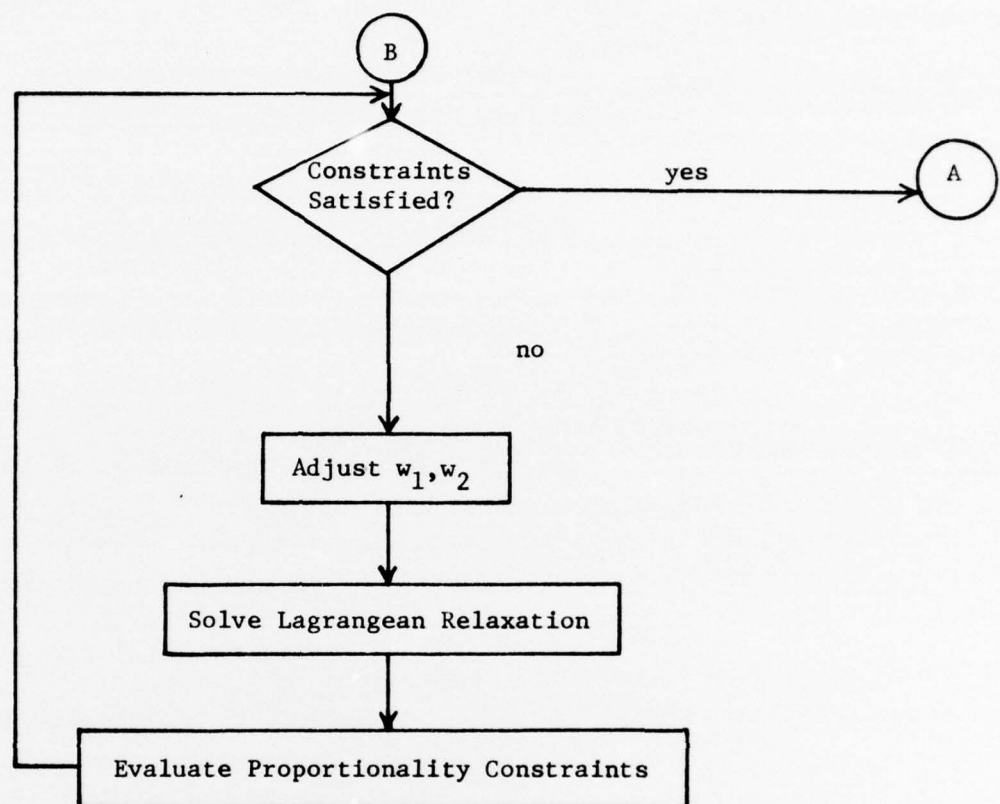
VII. FLOWCHART



# VIII. DESCRIPTION OF SUBROUTINES

## Routine Name

MAIN	Calling routine, sets parameter default values.
COMMAND	Processes all commands and issues calls to the other routines when appropriate.
REPORT	Produces printed output of problem solutions.
SETUP	Reads in arc data cards, generates arrays for a new problem to be solved, and writes cost, desirability, utility, node name, upper bound, and current composite objective function arrays on scratch file.
NODENO	Assigns node numbers to node names and checks for maximum number of nodes on input. Entry point NODNO returns node number for a previously assigned node name.
MODIFY	Modifies appropriate arc parameters to set up appropriate objective functions for various subproblems giving $c^*$ , $d^*$ , $u^*$ , and arising from the Lagrangean search procedure.
SIDE	Determine from a given arc's marginal cost and flow whether the arc is label eligible.
SETCMP	Initializes vectors for the problem in memory prior to solution. Entry point SOLVE applies the out-of-kilter method to solve the problem currently set up in memory.
RIGHT	Partitions arcs leaving a given node into label eligible and label ineligible subsets.
SCRIN	Using the "cost" arrays on the scratch file, set up the objective function for the next subproblem.
SCROUT	Restore the scratch file data.
SLACKS	Compute the values of slack variables for the constraints on functional deviations from optimality.
ARCNO	Find the arc number of an arc given in terms of from node and to node names.



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